Investigation of Stochastic Differential Models and a Recursive Nonlinear Filtering Approach for Fusion-Prognostics

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1. PROBLEM BEING ADDRESSED

Electronics-rich systems perform important societal functions in diverse fields. Failures in such systems can cause loss of revenue and lives, e.g., the Xian MA-60 propeller plane that crashed into sea leaving 27 people dead in May 2011, or the failure in a point-of-sale information verification system that can result in loss of sales worth $5,000,000/min. These failures and unplanned downtime could be prevented if these systems could be made self-cognizant, i.e., if they could self-assess performance, estimate their remaining useful life (RUL), and adaptively make decisions for mitigating risks. However, this has been difficult to achieve because of a lack of understanding of the interactions between system parameters and application environments and their effect on system degradation.

In order to address these issues, Pecht (Pecht, 2010; Cheng & Pecht, 2009) introduced the fusion prognostics approach. This approach first defines a degradation model that takes into account the presence of multiple failure mechanisms, varying environmental conditions, unit-to-unit uncertainty, and the uncertainty (temporal) associated with the progression of degradation. Existing degradation models for electronic systems simply aggregate the degradation of critical components and fail to consider the unit-to-unit variability. In these models environmental effects are mostly addressed from an accelerated testing perspective, which helps to analyze only a specific type of degradation (Gu, 2009; Alam, 2010; Kwon, 2010). Recent variants of these models that account for unit-to-unit uncertainty and the time-varying environmental effects have failed to account for temporal uncertainty (Gebraeel, 2008 & 2009). And few of the reported models consider degradation as a result of both wearout and overstress mechanisms (Kharoufeh, 2006; Shetty, 2008; Rangan, 2008).

Another challenge has been the identification of appropriate RUL estimation techniques. Recently, Unscented Kalman (Tian, 2011) and Particle Filters (Orchard 2007; Zio, 2011) have been receiving a lot of attention for this purpose (Saha, 2009). However, degradation is a continuous process, and these techniques require the degradation to be a discrete process. It is generally assumed that the degradation is Gaussian in nature and there exists a predetermined failure threshold. However, it has been proven that the degradation process can have a skewed distribution and that there does not always exist a predetermined threshold. Thus, modeling errors and assumptions about the degradation process contribute to uncertainty in estimation. Thus, there is a need for new techniques to improve the confidence level of RUL estimates. Also, it is desirable to make the RUL estimates in a recursive manner and identify the possible failure mechanisms from the predicted degradation state.

In this work, the focus is on developing a generic fusion prognostics approach that will allow systems to self-assess performance and recursively estimate RUL. For this generic fusion prognostics approach we have the following goals:

a) To define a mathematical model that best describes the dynamic nature of system degradation and
b) To construct a recursive algorithm for the defined model that uses only the previous estimate of the RUL and the latest observations to make a new estimate of the RUL.

Initially, we investigate the use of stochastic differential equations (SDEs) for modeling system degradation. The states of the model are defined using the parameters reflecting system response and their behavior with respect to time and usage. The effects of wearout and overstress mechanisms are included by decomposing the degradation process into two sub-processes. Based on the investigation, the algebraic and geometric structure of the SDE representing system degradation will be defined. For the autonomous functioning of the system, recursive and optimal nonlinear filtering equations are derived to estimate future health states. For computational feasibility, the finite dimensional form of the filtering equations will be derived from optimal filtering equations by using concepts from nonlinear systems, the theory of Lie algebra, and recent insights gained from particle filters.

2. EXPECTED CONTRIBUTIONS

2.1 Modeling System Degradation

One goal of this research work is to construct and evaluate a degradation model whose state, $X \in \Re^n$, is defined using the system parameters and their behavior

(∂x and $\partial^2 x$) with respect to time and usage. SDEs arise when physical processes evolve as an ordinary differential equation is subjected to environmental fluctuations. We propose to model system degradation in two stages. In the first stage, we describe degradation as an SDE driven simultaneously by a jump and Wiener process. Defining degradation as a stochastic process accounts for the temporal uncertainty. Modeling the degradation as a jump-diffusion model allows us to decompose the degradation into two sub-processes: a continuous diffusion process and a discrete jump process. The continuous diffusion process represents the degradation due to wearout mechanisms, and the discrete jump process accounts for the degradation caused by overstress mechanisms, respectively.

$$dx_t = \underbrace{f(x_t,\Theta)\cdot dt + g(x_t,\Theta)\cdot dw_t}_{wearout} + \underbrace{\int_{\Re^l} \gamma(x_{t^-}, z)\cdot \mathrm{P}(ds,dz)}_{overstress} \tag{1}$$

In the second stage, this model is transformed into a random coefficient model consisting of deterministic and stochastic coefficients. The deterministic coefficients will capture degradation characteristics that are fixed across a population of components, thereby allowing us to incorporate the knowledge of physical degradation. The stochastic coefficients that are estimated from condition monitoring will account for the degradation due to the time-varying nature of the prevailing environment.

Modeling the state using performance parameters and their behavior (derivatives) with respect to time and usage will help us in defining the subspace (in the entire state space) occupied by the failure states (corresponding to a failure mechanism) using the information obtained from degradation testing. Here, the state space includes all possible values taken by the performance parameters and their derivatives during both system failure and normal operation. This knowledge will help in identifying the possible failure mechanisms based on the evolution of degradation.

Based on the investigation, the algebraic and geometric structure of the SDE representing system degradation will be defined. Parameter estimation techniques will be investigated for re-estimating the model parameters once new observations are available.

2.2 Recursive Nonlinear Filtering

In order to predict, detect, and localize failure, it is critical to forecast the health state of a system. Thus, the second goal is to address the nonlinear filtering problem, which concerns the Bayesian estimation of a health-state process, $\{\mathrm{x(t)}\}$, based on discrete observations, $\{\mathrm{y(t_k)}\}$. It is preferable to perform computations recursively in terms of a statistic, $\theta = (\theta_t)$ that can be updated using the latest observations. By doing so, we can make the system function autonomously. Generally, this statistic, $\theta = (\theta_t)$, is related to the conditional distribution of $\{\mathrm{x(t)}\}$, given the past observations. Thus, we require computations of the form

$$\begin{aligned} &E\left[x(t) \mid y(s_k) : 0 \le s_k \le \tau\right] \\ &= \beta\left(y(\tau), E\left[x(\tau) \mid y(s_k) : 0 \le s_k \le \tau - 1\right]\right) \end{aligned} \tag{2}$$

where, $\boldsymbol{\beta}$ represents a nonlinear function of the previous RUL estimate and the latest observations.

However, the presence of the jump process makes the optimal filtering equations infinite in dimensions. Thus, the third goal is to define a finite dimensional sub-optimal recursive filter based on the optimal filtering equations. The structure of a recursive nonlinear filtering problem leads naturally to the use of methods from nonlinear system theory and the theory of Lie algebras (Marcus, 1978; Marcus 1984). The application of these methods to nonlinear filtering problems has led to a number of results related to finite dimensional filters. Hence, necessary generalization will be proved and coupled with the theory of Lie algebra to obtain finite dimensional filters. Also, the use of particle filters will be investigated for projecting and estimating the probability density function (PDF) of the RUL. This is because the particle filtering framework can prove transformative in understanding the evolution of the RUL PDF based on Kolmogrov equations.

2.3 Approach Validation

In order to validate the proposed approach, we will demonstrate the approach on power systems using Insulated Gate Bipolar Transistors (IGBTs). Power systems using IGBTs are often used in aerospace applications where high operational and environmental stresses can cause rapid failure of critical components. A physics of failure approach will be used to identify the state of the degradation model. The subspace occupied by different failure mechanisms will be obtained from life-cycle testing. For conformance of theoretical results to experimentation, power systems will also be subjected to field operation, and condition monitoring will be performed. The in-situ degradation attributes and the environmental conditions monitored will be used to estimate the stochastic coefficients of the model. Finally, the recursive filter will aid in determining the RUL of the system.

3. PROPOSED PLAN

After an initial literature review, a feasibility study of the degradation models considered was carried out. Recursive and optimal filtering equations for the SDE containing only the stochastic coefficients were derived. Our current work involves the validation of these filtering equations through simulations based on Euler's approximation method. Future work involves the derivation of finite dimensional filtering equations. Then, an SDE model with a defined algebraic and geometric structure has to be constructed. Appropriate parameter estimation techniques have to be investigated to estimate model parameters in real time. Further, optimal and sub-optimal filtering equations will be derived for predicting system degradation. Simultaneously, at each stage the approach will be validated using the power system.

4. PRELIMINARY RESULT

We have simulated a one-dimensional SDE subjected to diffusion to represent the degradation path of a component. The estimation is obtained using nonlinear optimal filtering equations. The simulations are performed using Euler's approximation method.

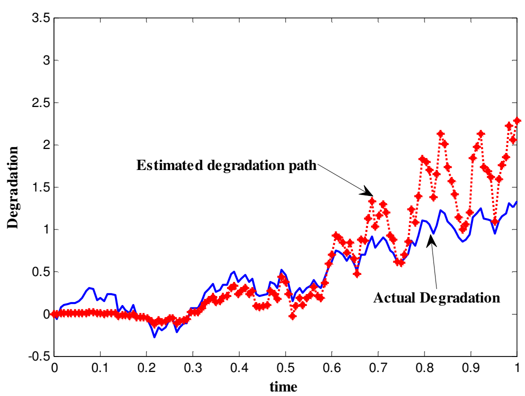

Figure 1: Simulation of degradation path using a one-dimensional SDE subjected to normal wearout mechanisms alone and the estimation of the same using Euler's approximation method.

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