



Machine Diagnostics using Advanced Signal Processing

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Presentation Layout

- **Background to separation of measured response signals – machine diagnostics and operational modal analysis**
- **Introduction to the cepstrum**
- **First separation – discrete frequency from stationary random and cyclostationary random components, including use of cepstrum, and application to bearing and gear diagnostics**
- **Second separation – forcing functions from transfer functions, including use of cepstrum**
- **Conclusion**



Separation of measured response signals

Two important situations in which one only has access to response signals are:

- 1. machine condition monitoring (MCM), where a change in condition could be indicated by a change in either the forcing function or structural properties**
- 2. Operational modal analysis (OMA), where one seeks to extract structural dynamic properties in the presence of forcing function effects. Also useful in machine diagnostics.**



INTRODUCTION TO THE CEPSTRUM

The cepstrum is defined as the inverse Fourier transform of a logarithmic spectrum, itself the forward Fourier transform of a time signal. Thus:

$$C(\tau) = \mathfrak{F}^{-1}[\log(X(f))] \quad (1)$$

where:

$$X(f) = \mathfrak{F}[x(t)] = A(f)\exp(j\phi(f)) \quad (2)$$

so that:

$$\log(X(f)) = \ln(A(f)) + j\phi(f) \quad (3)$$

The abscissa τ of the cepstrum has the dimensions of time but is known as “quefreny”. If the data is sampled the Fourier transforms can be replaced by Z-transforms



TYPES AND PROPERTIES OF CEPSTRUM

- If phase is retained in the log spectrum, the cepstrum is called the “complex cepstrum” (despite being real)
- The complex cepstrum is reversible to a time signal but requires continuous unwrapped phase
- Real stationary signals with noise and discrete frequencies do not have continuous phase
- If phase is discarded, the “real cepstrum” or “power cepstrum” is obtained - the latter can be based on an averaged power spectrum
- Cepstrum has “rahmonics” corresponding to families of harmonics and sidebands in the log spectrum



CEPSTRUM TERMINOLOGY

SPECtrum	→	CEPStrum
FREQUency	→	QUEFRency
HARmonic	→	RAHmonic
MAGnitude	→	GAMnitude
PHASe	→	SAPHe
FILter	→	LIFter
Low pass filter	→	Short pass lifter
Frequency analysis	→	Quefrequency alanalysis

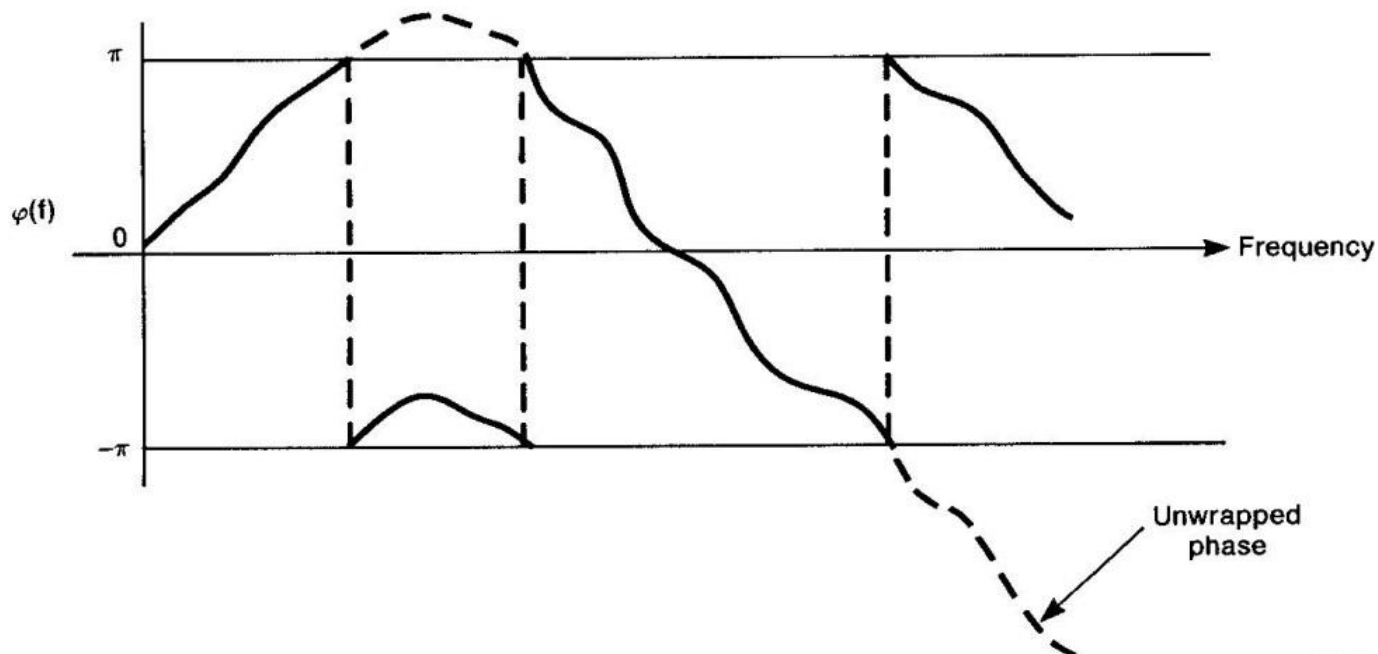
Ref: Bogert, Healy and Tukey (yes the one of FFT fame, but two years earlier) – “The Quefrequency Alanalysis of Time Series For Echoes; Cepstrum, Pseudo-autcovariance, Cross-cepstrum and Saphe Cracking”. Proc. Symp. On Time Series Analysis, Wiley, 1963.

Complex Cepstrum

$$C(\tau) = \mathfrak{F}^{-1}[\log(X(f))]$$

where $X(f) = \mathfrak{F}[x(t)] = A(f)\exp(j\phi(f))$

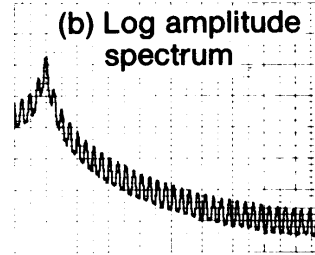
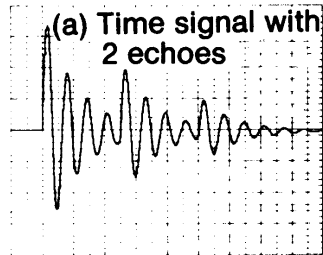
BUT phase must be a continuous function of frequency, ie “unwrapped”



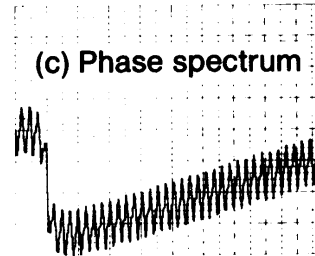
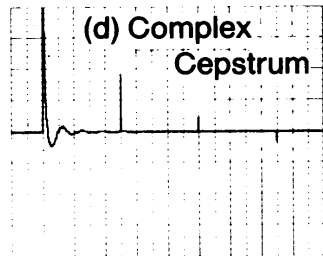


ECHO REMOVAL USING THE CEPSTRUM

Echoes overlap
original signal

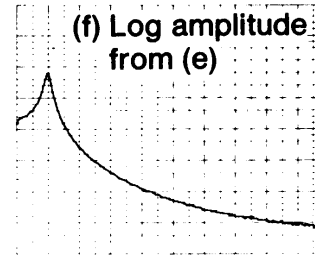
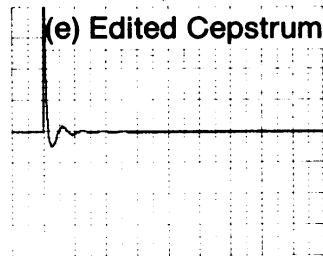


Echoes give delta
functions in cepstrum

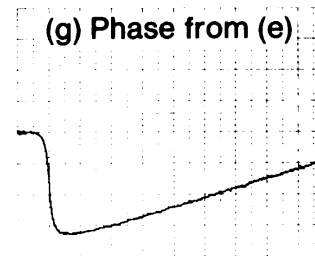
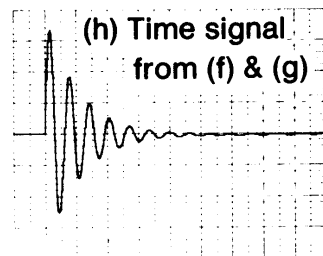


Echoes give added periodic
function in log amplitude
and phase spectra

Delta functions
removed



Overlapping echoes
removed



Smoothed log amplitude
and phase



APPLICATION OF CEPSTRUM TO MACHINE DIAGNOSTICS

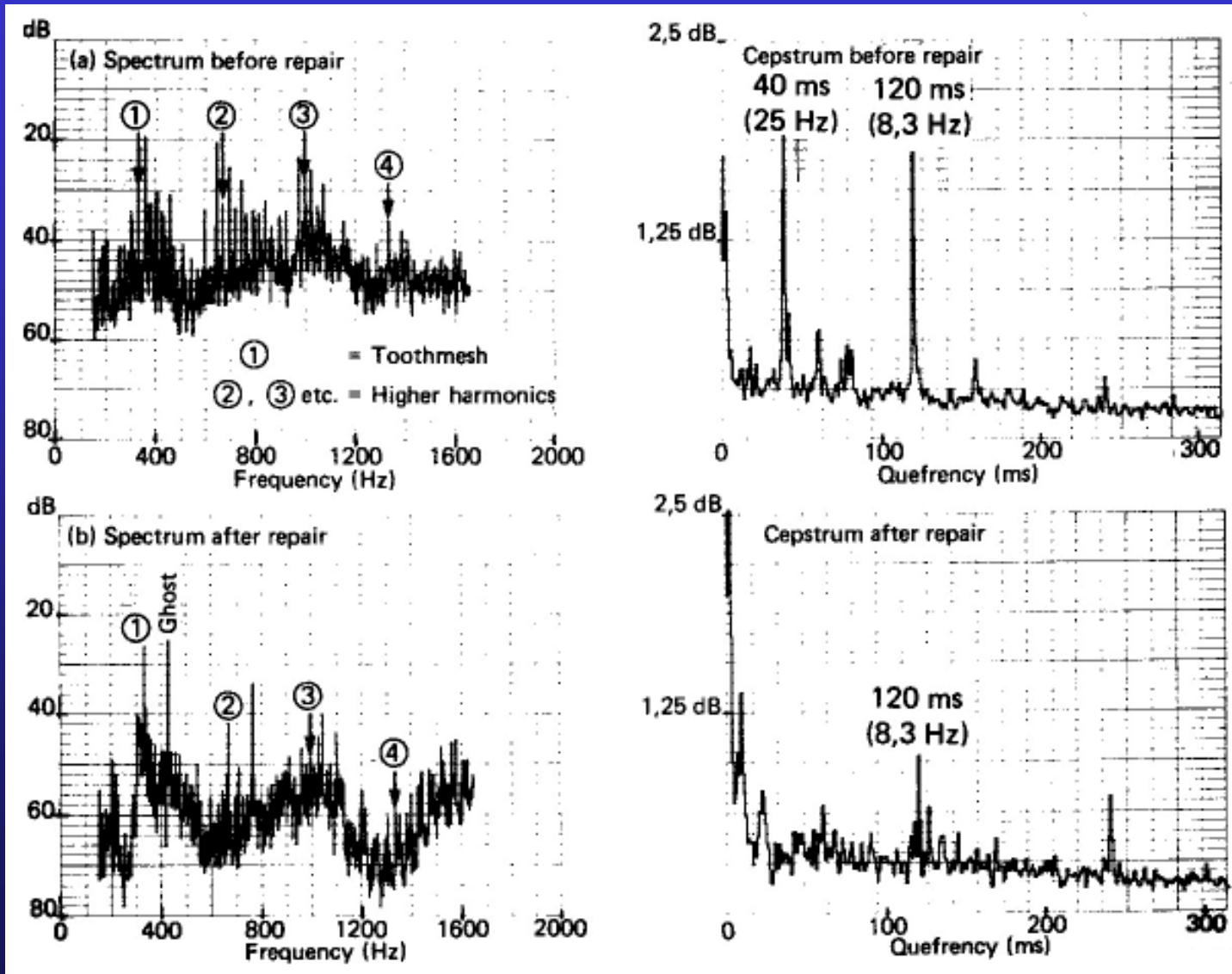
A. Detection of periodic structure in spectrum

- Harmonics (Faults in gears, bearings, blading)
- Sidebands (Faults in gears, bearings, blading)
- Echoes, reflections

B. Separation of Source and Transmission Path Effects” (SIMO)



USE OF CEPSTRUM FOR SIDEBAND PATTERNS

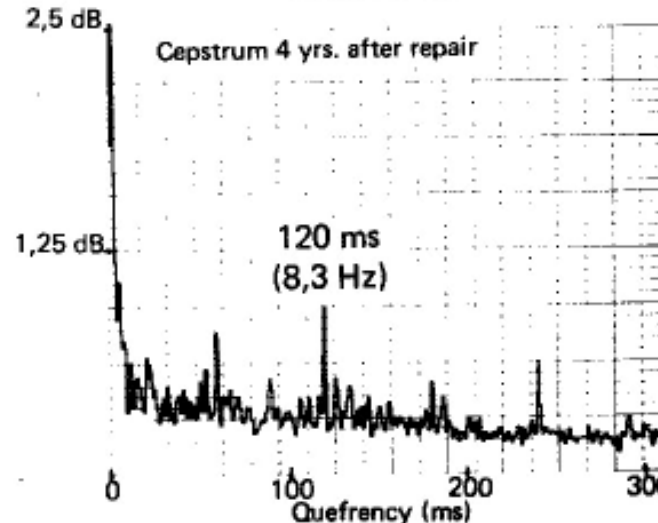
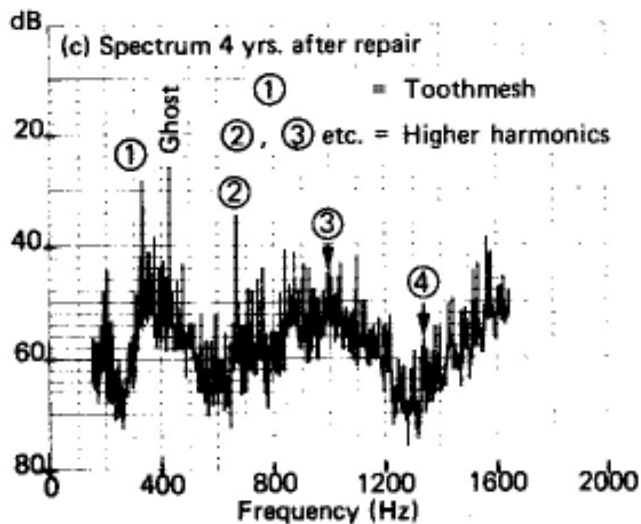
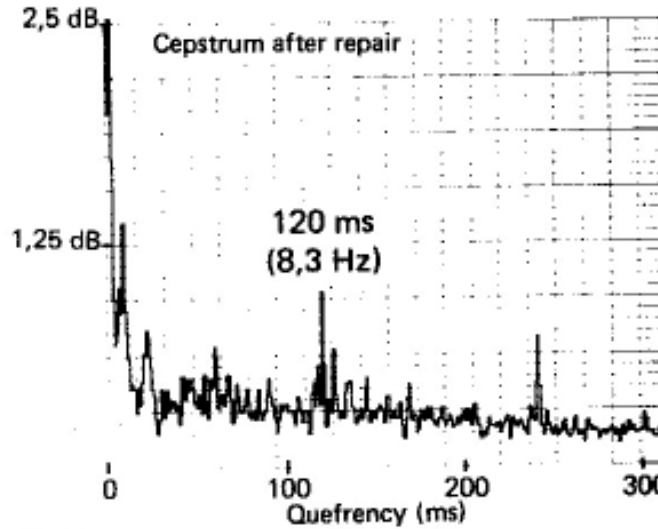
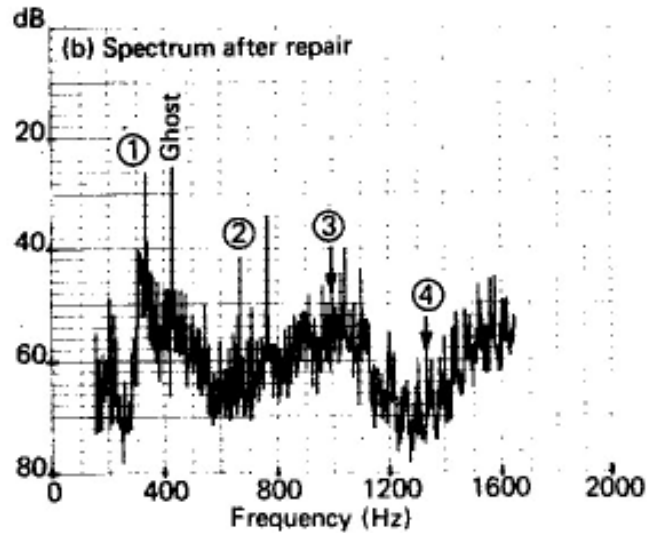


2 families of sidebands – triangular wear pattern due to lapping

Initially smaller sidebands, only at gear speed



LATER DEVELOPMENT

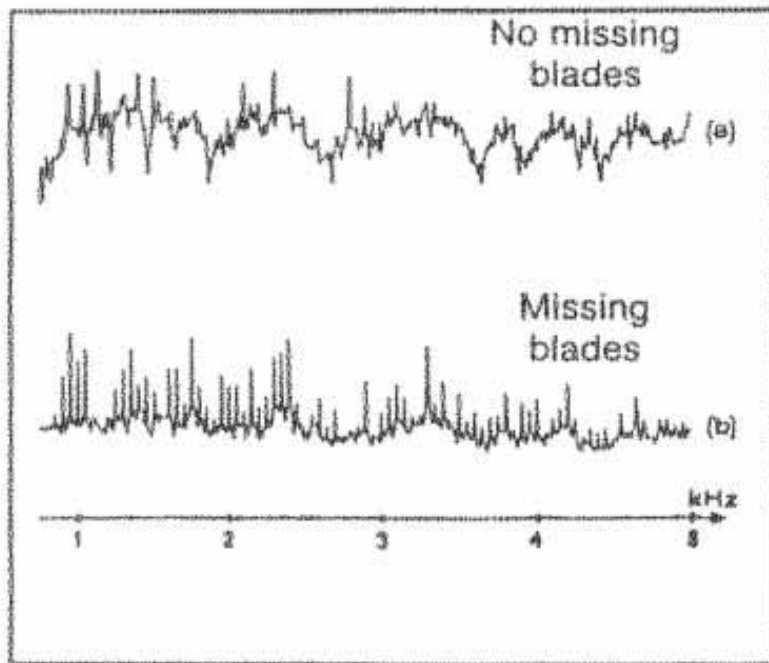


Original
condition

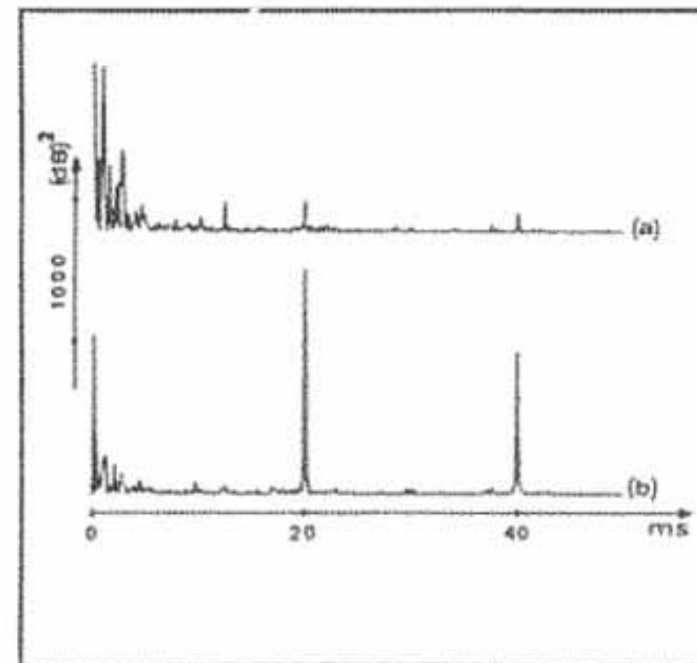
After 4 years, 2nd
harmonic of
garmesh has
increased and 2nd
ghost component
reduced
(indicating wear),
but no further
sideband growth

Use of cepstrum to detect missing blades in a steam turbine (French Electrical Authority EDF)

Power Spectra



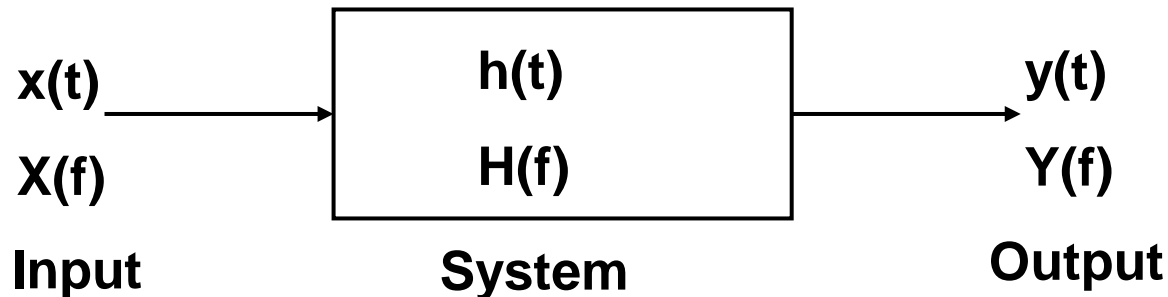
Cepstra



Missing blade causes misdirected steam jet to impinge on local stator area once per rev; picked up by casing mounted accelerometer. Increased shaft speed harmonics in mid frequency range.



Separation of Source and Transmission Path Effects (SIMO only)



$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f) \cdot H(f)$$

$$|Y(f)|^2 = |X(f)|^2 \cdot |H(f)|^2$$

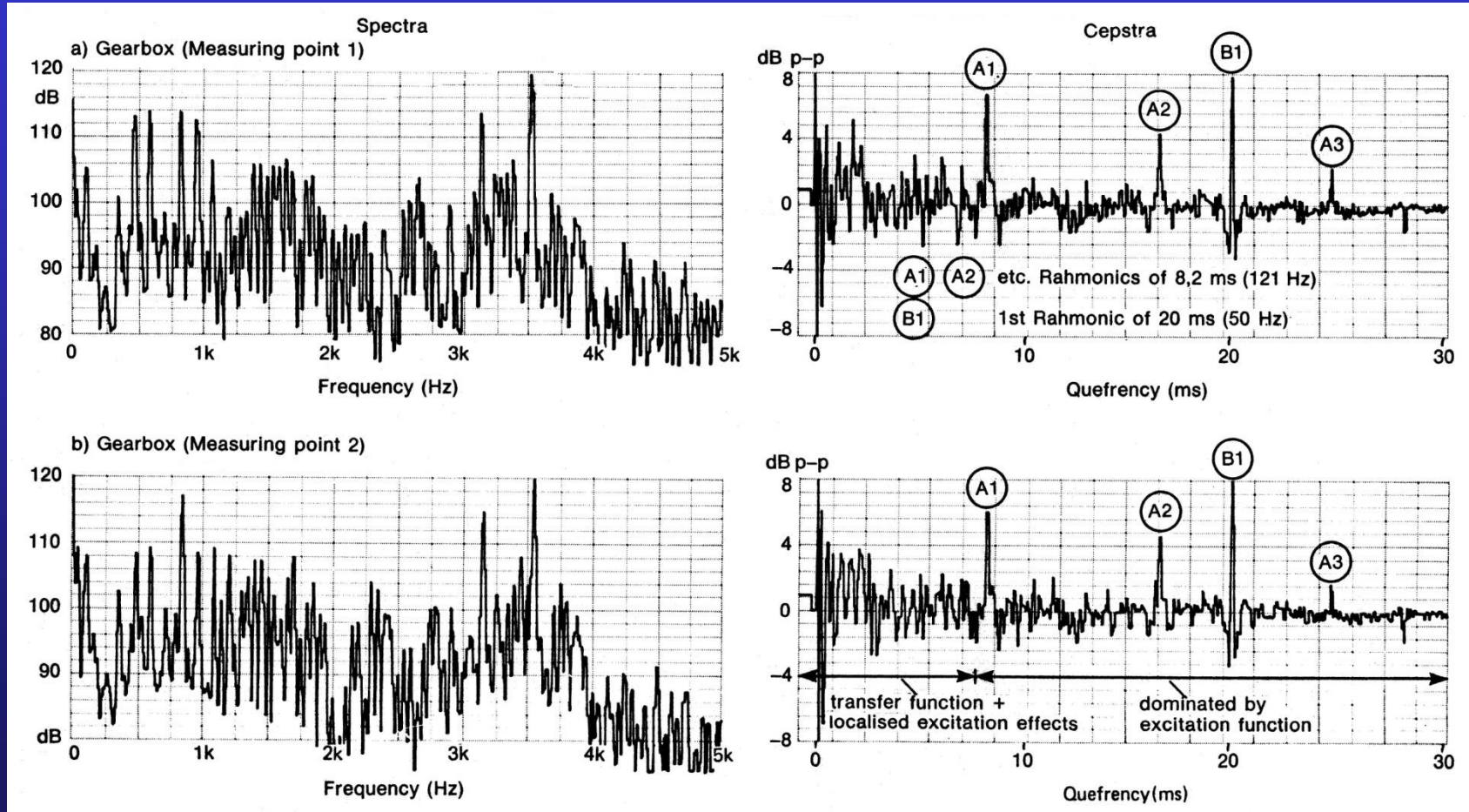
$$\log Y = \log X + \log H$$

$$\mathfrak{F}^{-1}\{\log Y\} = \mathfrak{F}^{-1}\{\log X\} + \mathfrak{F}^{-1}\{\log H\}$$

Thus, source and transmission path effects are additive in cepstrum. Moreover, they are often separated



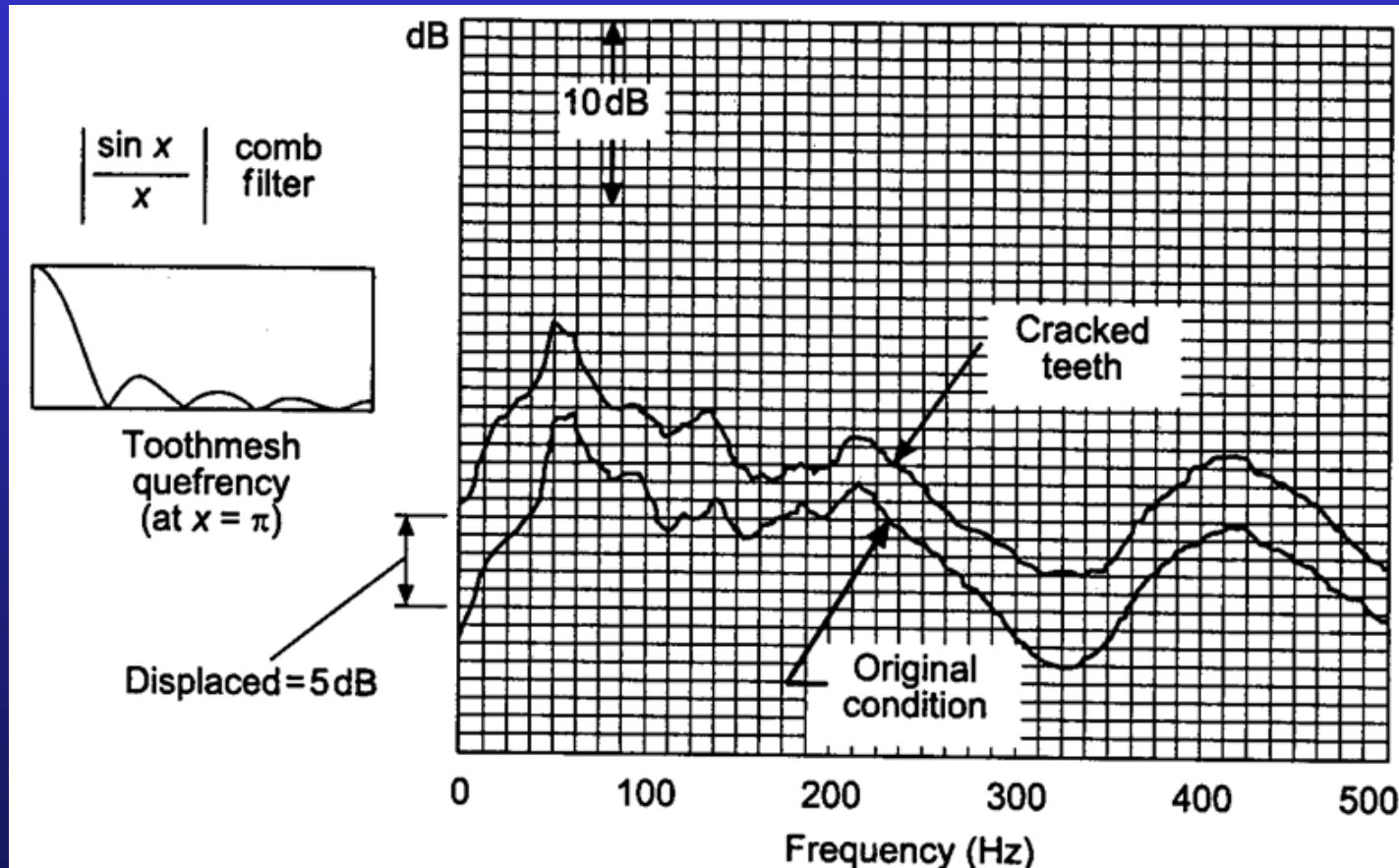
INSENSITIVITY OF CEPSTRUM TO TRANSFER PATH





Use of Cepstrum to check Transfer Function

Forcing function component removed from the low frequency part of the cepstra for a gear with and without cracked teeth



Unchanged resonances confirm that change is at source



BACKGROUND TO CM

- **Most condition monitoring involves separation of signals from different sources**
- **A typical case is separation of gear signals from bearing signals in a gearbox**
- **Gear signals are deterministic (when tooth contact maintained)**
- **Bearing signals are stochastic because of random slip**
- **This permits their separation, even when the gear signals are much stronger**



Methods for the Separation **PHM Montreal** of Deterministic and Random Signals

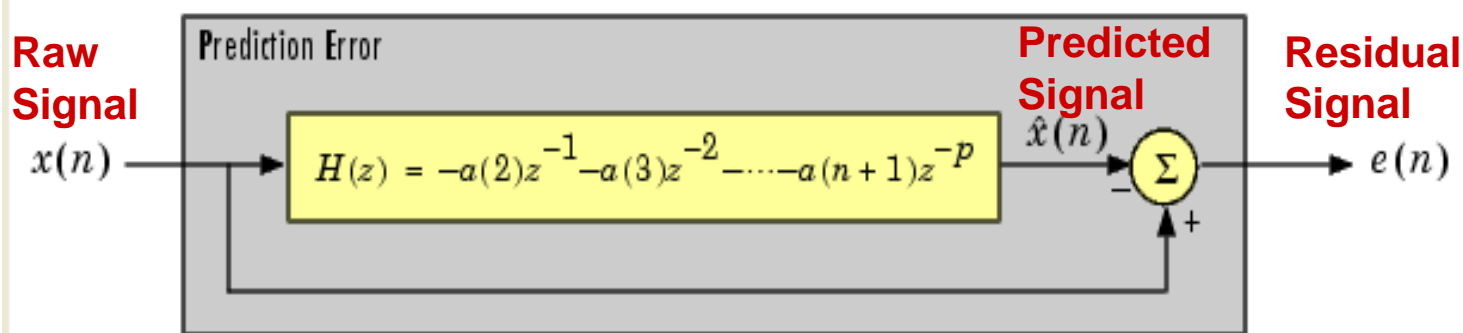
- **Linear prediction** - gives simultaneous prewhitening. Some choice of what is removed by order of filter.
- **Self adaptive noise cancellation (SANC)** - copes with some speed variation. Removes all deterministic components.
- **Discrete/random separation (DRS)** – more efficient than SANC, but may require order tracking. Removes all deterministic components.
- **Time Synchronous Averaging (TSA)** – minimum disruption of residual signal – requires separate angular sampling for each harmonic family – Does not remove modulation sidebands.
- **New cepstral method** – removes selected uniformly spaced frequency components, including sidebands – Can leave some if required.



Autoregressive (AR) model used for Linear Prediction

1. In an Autoregressive model (AR), we try to capture the information about the deterministic part using linear prediction.
2. The value Y for sample number n is expressed as a linear combination of previous p elements, i.e

$$Y_n = \underline{a(2)}Y_{n-1} + \underline{a(3)}Y_{n-2} + \underline{a(4)}Y_{n-3} + \dots + \underline{a(p+1)}Y_{n-p}$$



Residual signal is “whitened” (noise and impulses)

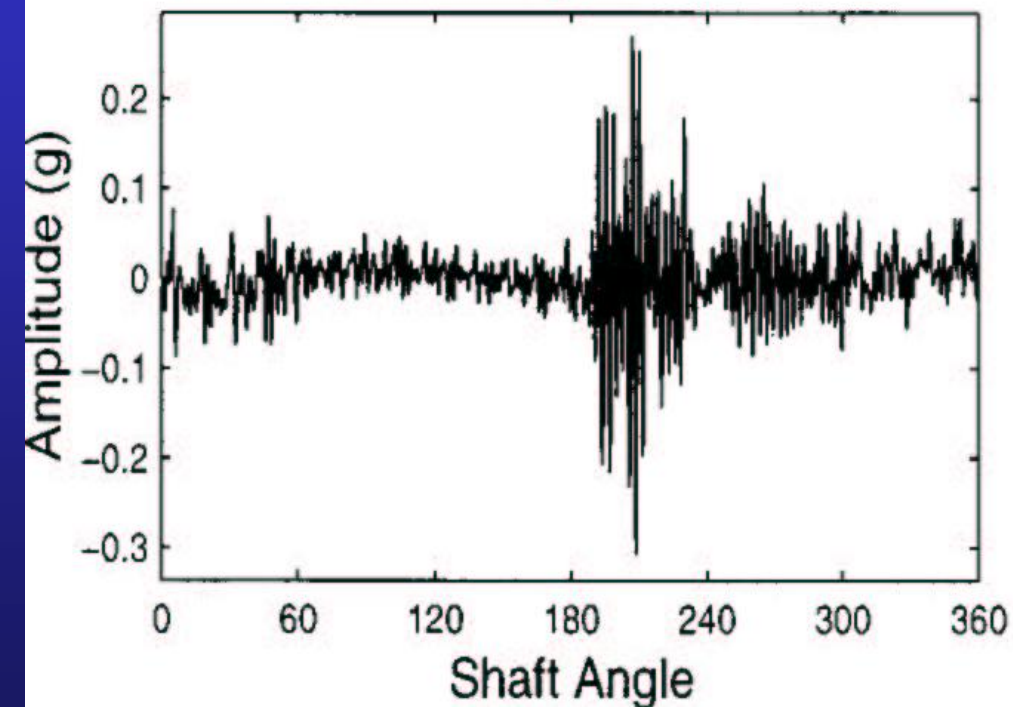


Residual Analysis of local gear faults by Linear Prediction

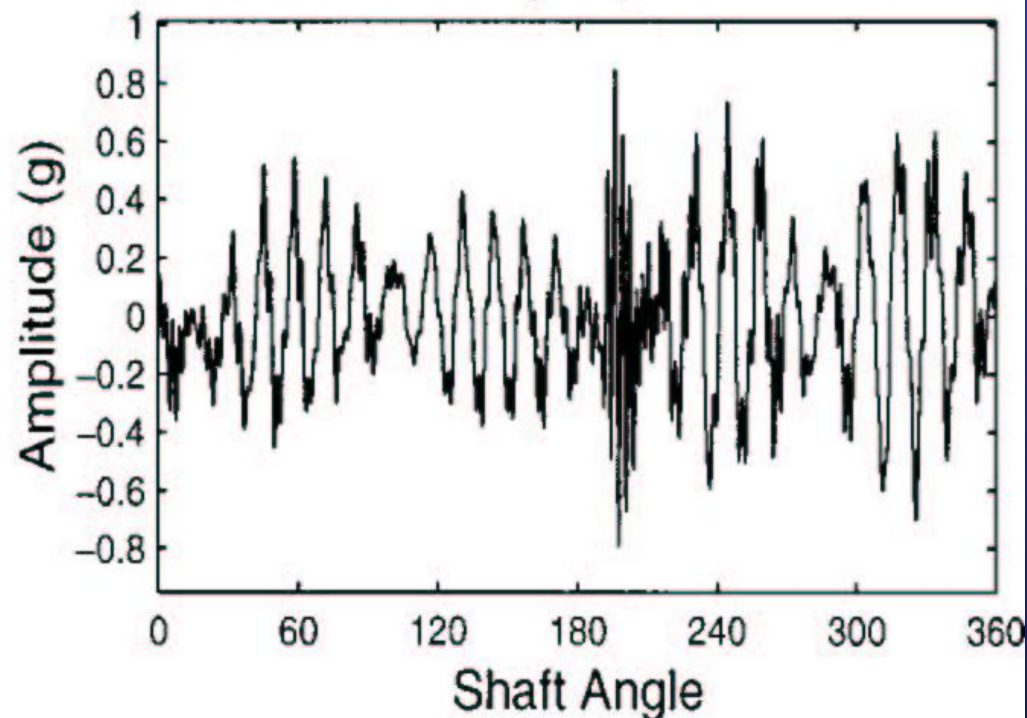
AR Method

Conventional Method
(removal of toothmesh harmonics)

(c) AR residual signal, kurtosis: 11.14



(d) residual signal, kurtosis: 3.03



W. Wang and A. K. Wong (2002) "Autoregressive Model-Based Gear Fault Diagnosis",
Trans. ASME, Journal of Vibration and Acoustics, 124, pp. 172- 179.



SANC

(Self Adaptive Noise Cancellation)

ANC requires *Two Inputs*:

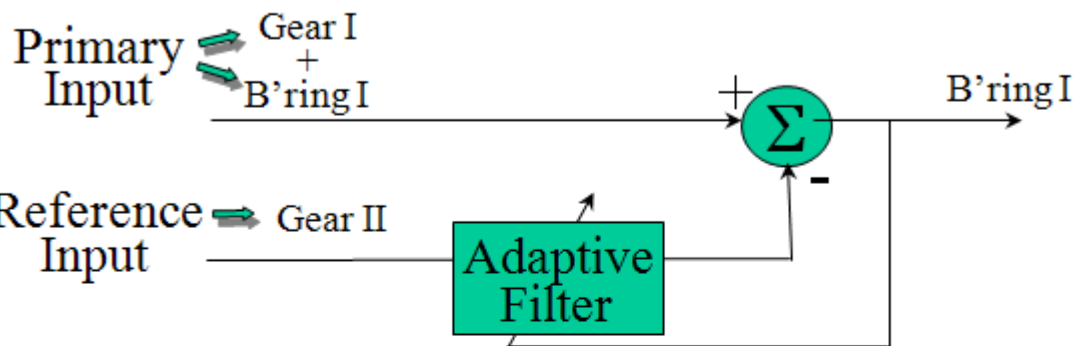
- Primary measured on a bearing housing;
- Reference measured far away from the bearing housing.

Adaptive filter compensates for transfer function between Gear I and Gear II
 SANC uses fact that bearing signal has a short correlation length

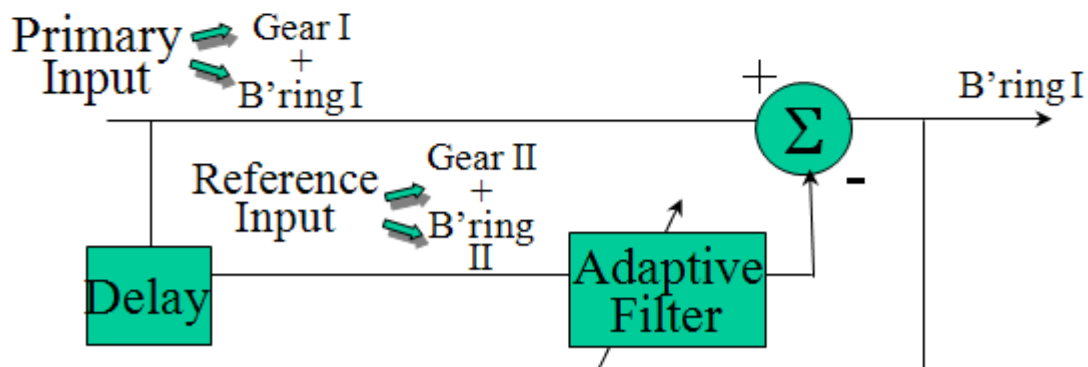
SANC requires *One Input*:

- Hard to get a reference, e.g. planetary bearing faults;
- Reference is the delayed primary;
- Applied to the bandpass filtered time signal in the envelope analysis procedure.

ANC

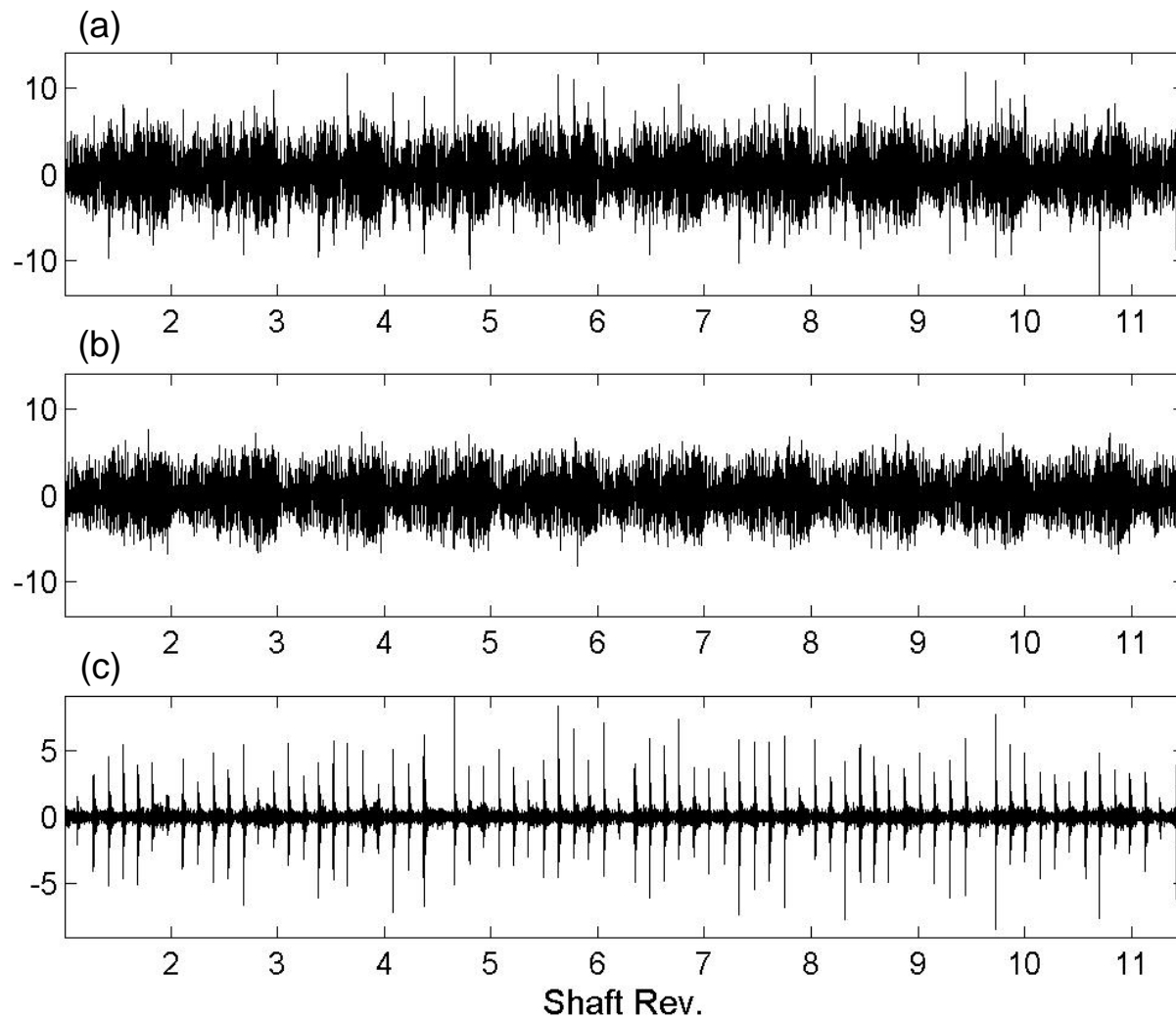


SANC





SEPARATION USING SANC



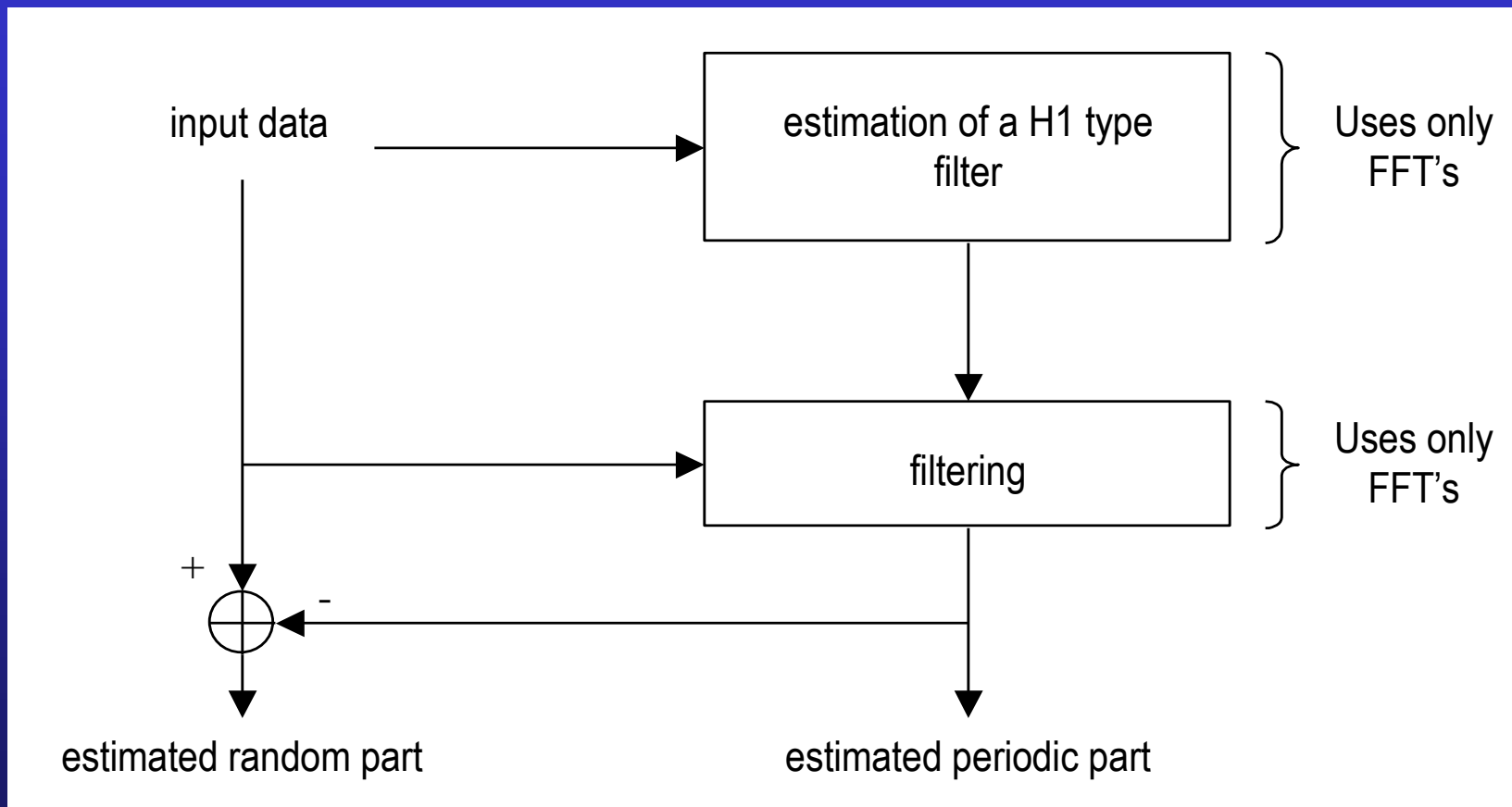
Signal from rig
(normal gear
signal with
bearing fault)

Gear signal
(discrete
frequency)

Bearing outer
race fault signal
(stochastic)



NEW METHOD OF DISCRETE - RANDOM SEPARATION



$$H_1 = \frac{G_{xy}}{G_{xx}} \text{ where } g_y(t) \text{ is a delayed version of } g_x(t).$$

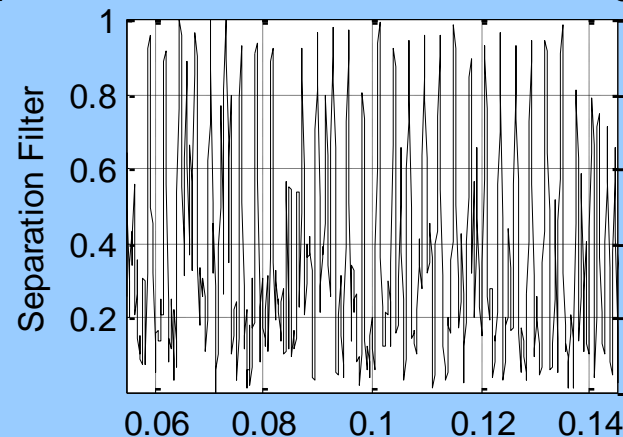
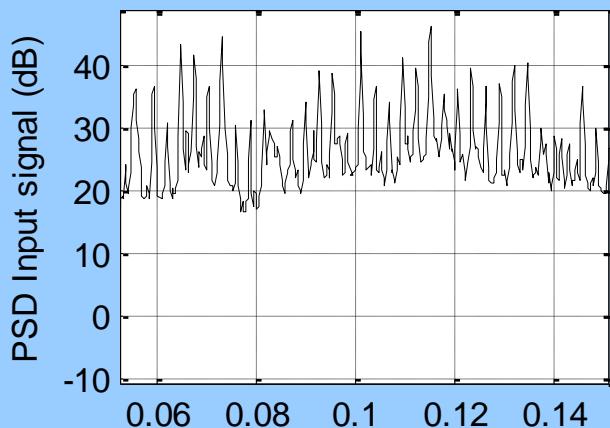
G_{xy} is the cross spectrum and G_{xx} is the autospectrum



DRS applied to a helicopter gearbox signal

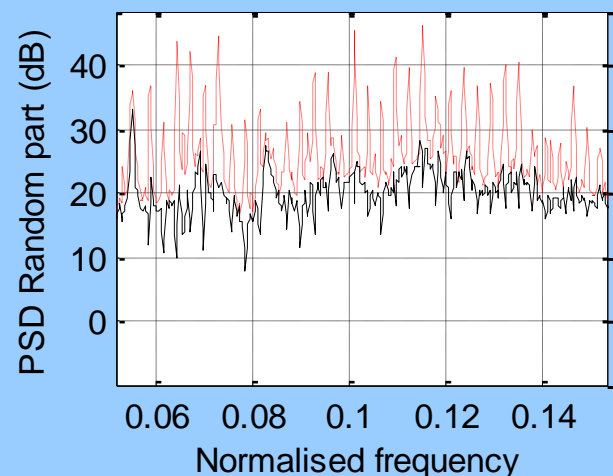
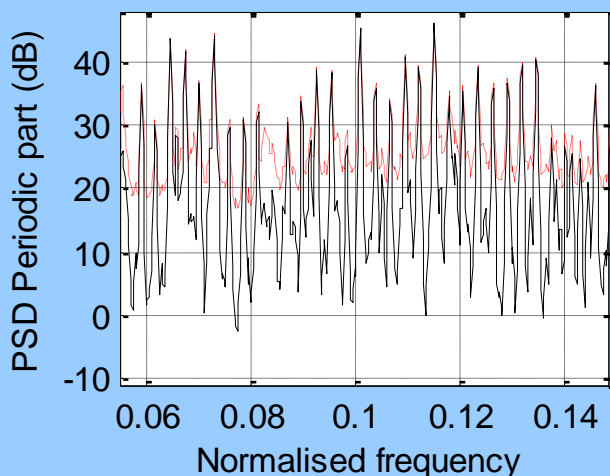
Time delay = 50; Filter length = 4096; Overlap = 50%; Window = Parzen; Resolution = single

Input
spectrum



Generated
filter
discrete = 1
noise = 0

Discrete
frequency
part



Noise part
by
subtraction



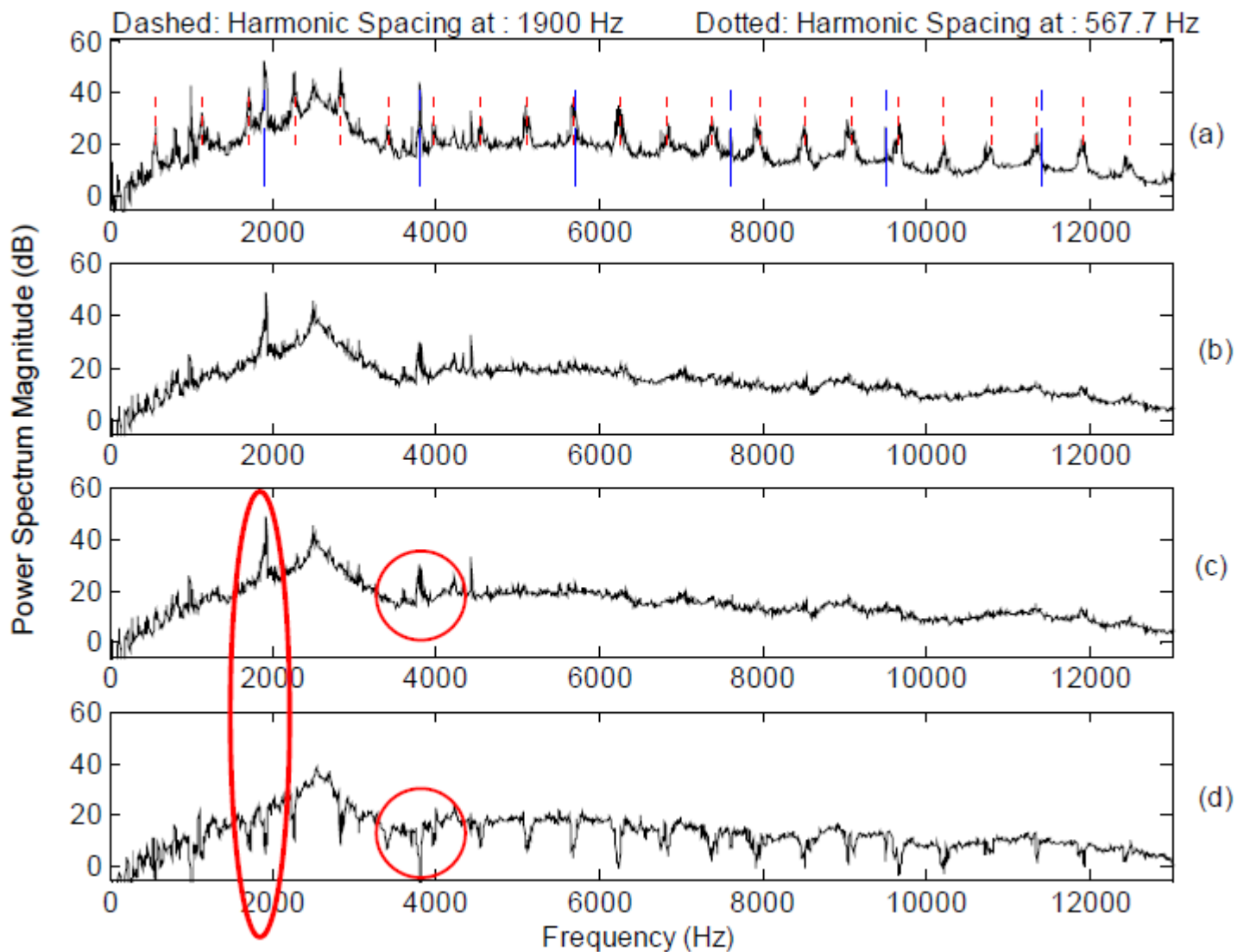
TIME SYNCHRONOUS AVERAGING (TSA)

- **Before TSA, signal must be order tracked to give integer number of samples per revolution and defined start point:**
- **One sample spacing corresponds to 360° of phase at sampling frequency and 140° of phase at highest valid frequency**
- **Just 0.1% speed fluctuation gives extra sample in typical 1024-point time record**
- **Sampling frequency may have to be changed for each gear in the signal**
- **Only removes harmonics – not sidebands**



COMPARISON OF TSA WITH DRS

(N. Sawalhi & R.B. Randall – CM-MFPT Edinburgh 2008)



Original
spectrum

TSA
Method 1

TSA
Method 2

DRS

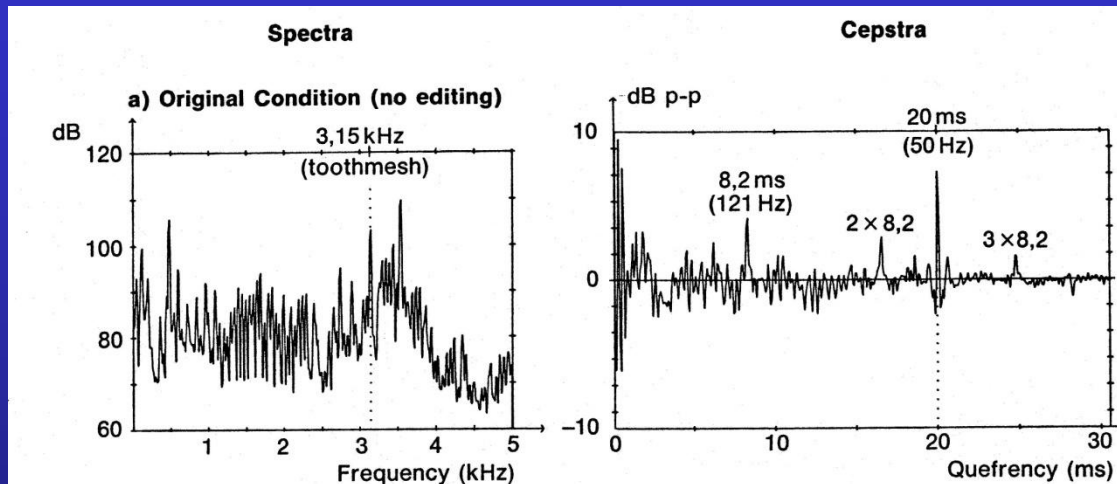


Editing Cepstrum

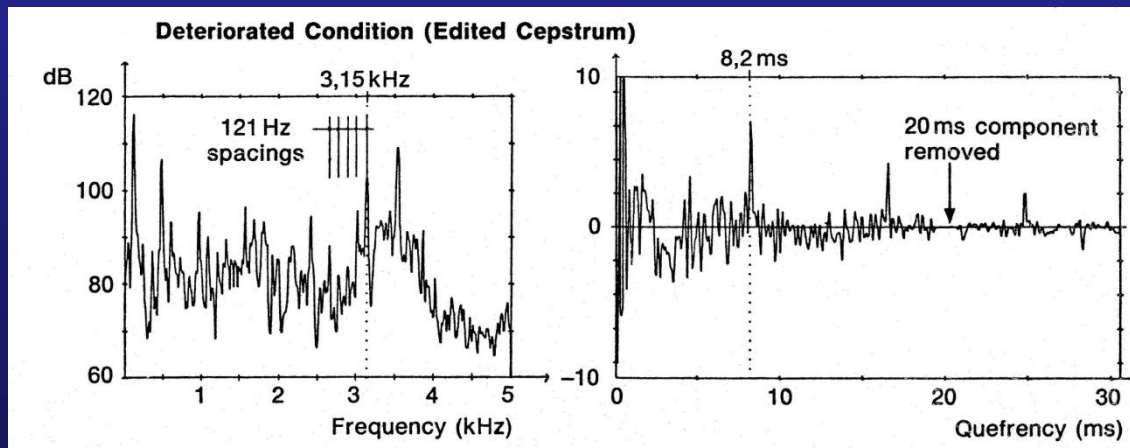
- Previously thought it was necessary to use Complex Cepstrum to edit time signal, eg echo removal
- Not possible to unwrap phase of excitation or response signals, therefore complex cepstrum excluded
- Real cepstrum used to edit spectrum, eg remove particular harmonic/sideband families, or reveal system resonances
- New proposed method uses the real cepstrum to edit the amplitude of force or response signals and combines with original phase to generate edited time signals



Editing cepstrum to remove specrum components



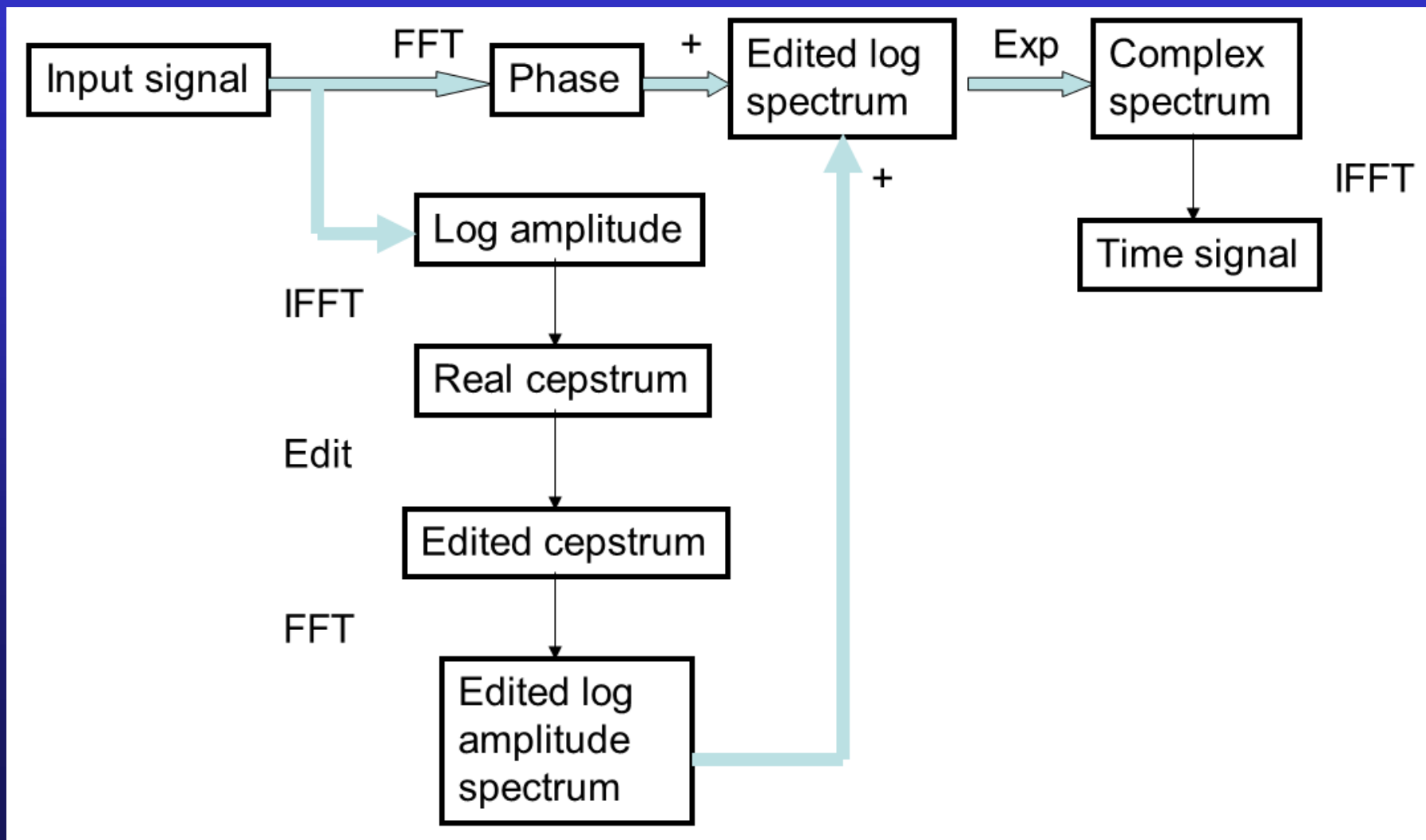
Original baseband
spectrum



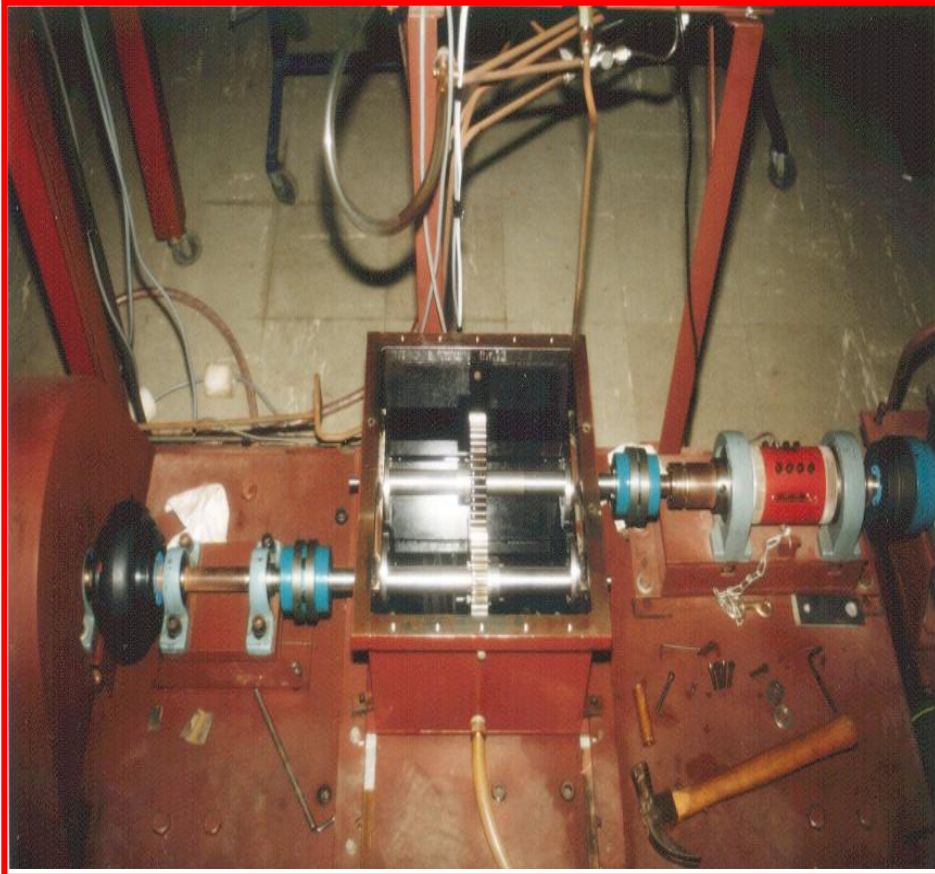
All harmonics (+ sidebands)
of 50 Hz shaft removed by
editing the 20 ms rahmonics
from the cepstrum and
forward transforming to the
log spectrum



NEW CEPSTRAL METHOD



Application to UNSW Gearbox Rig



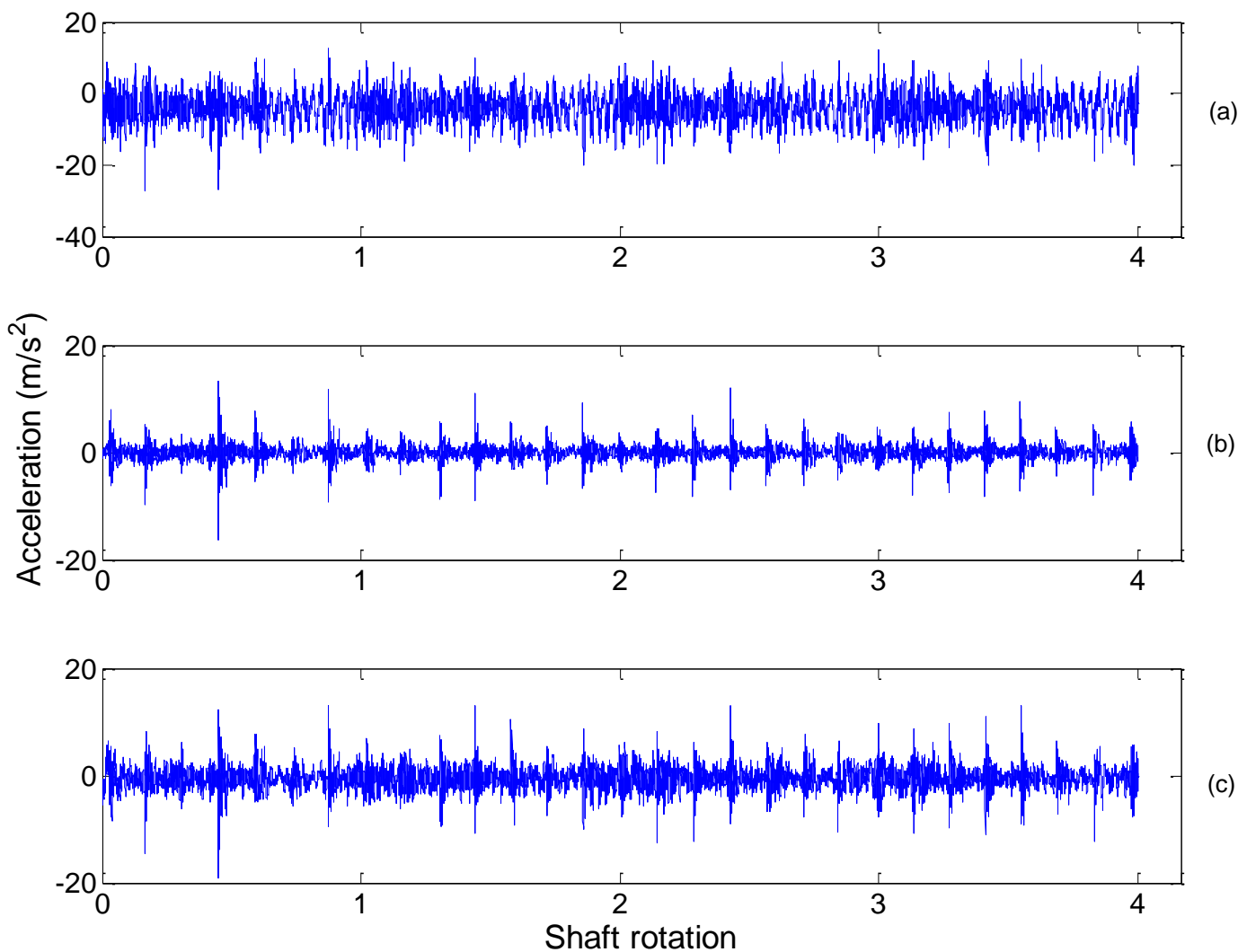
UNSW Spur Test Rig



Inner race fault



Time Domain Signals



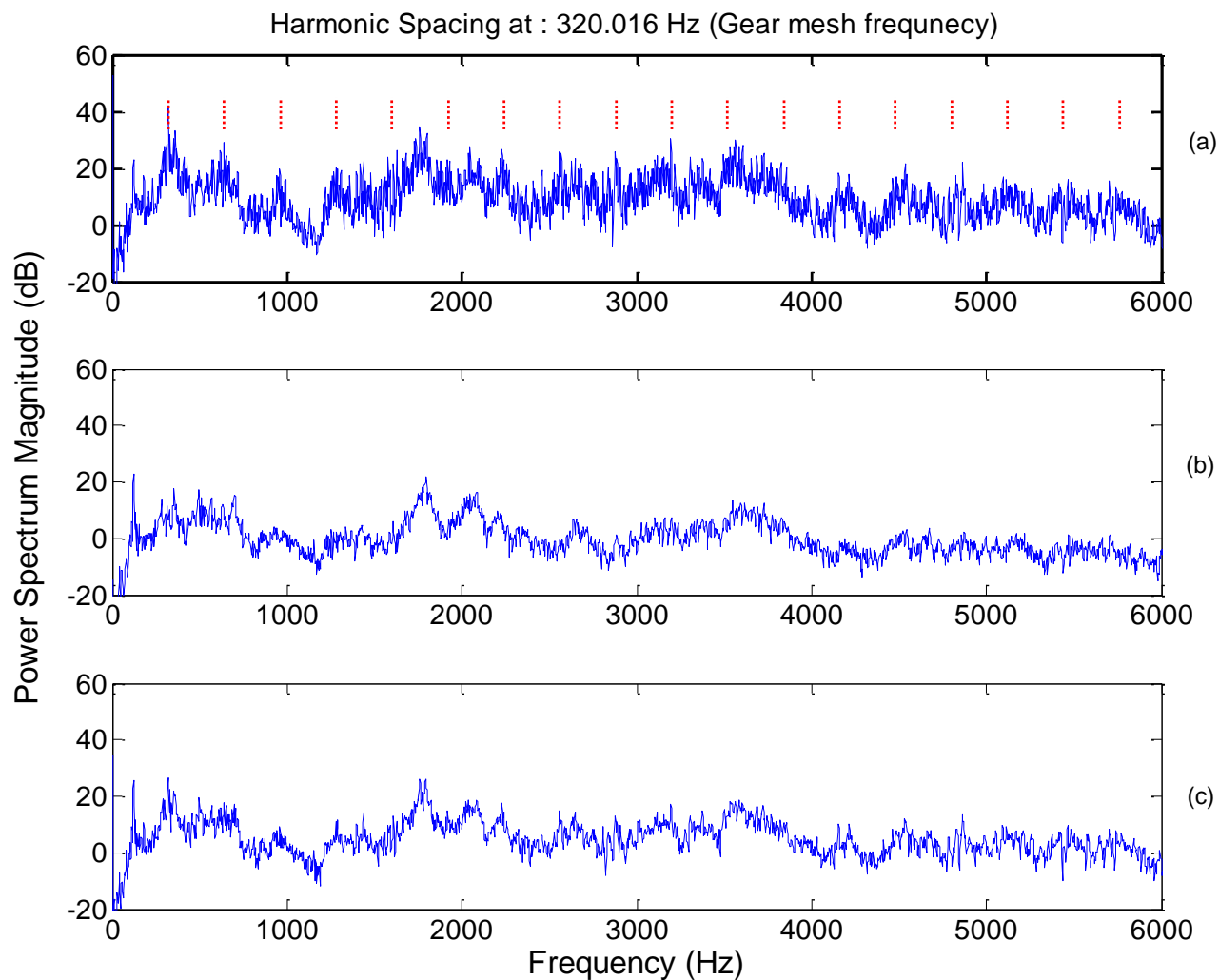
Raw signal

*Residual signal
(after removing
synchronous
average)*

*Residual signal
after editing
the Cepstrum*



Power Spectra



Raw signal

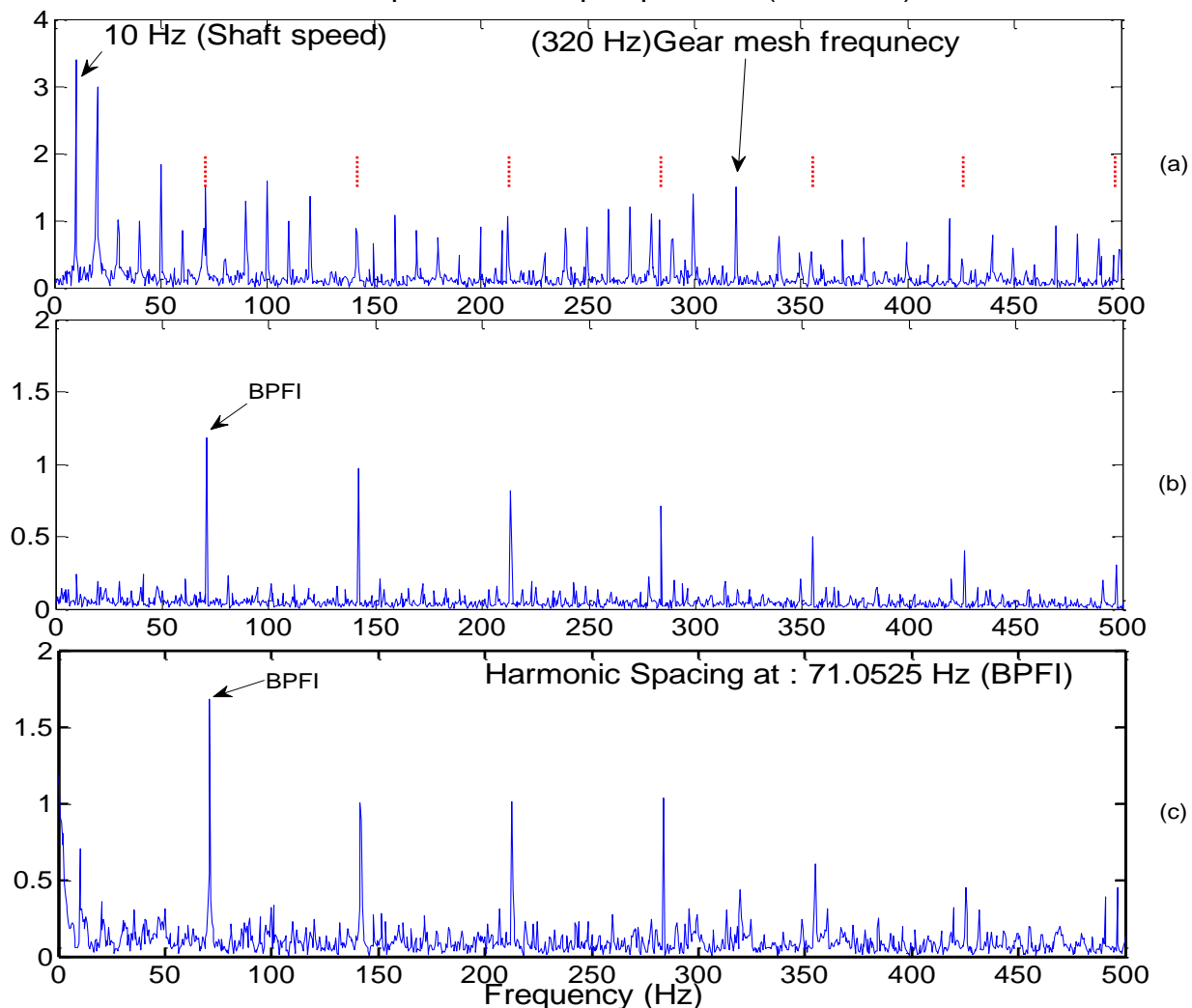
*Residual signal
(after removing
synchronous
average)*

*Residual signal
after editing
the Cepstrum*



Envelope Spectra

Squared envelope spectrum (1-20 kHz)



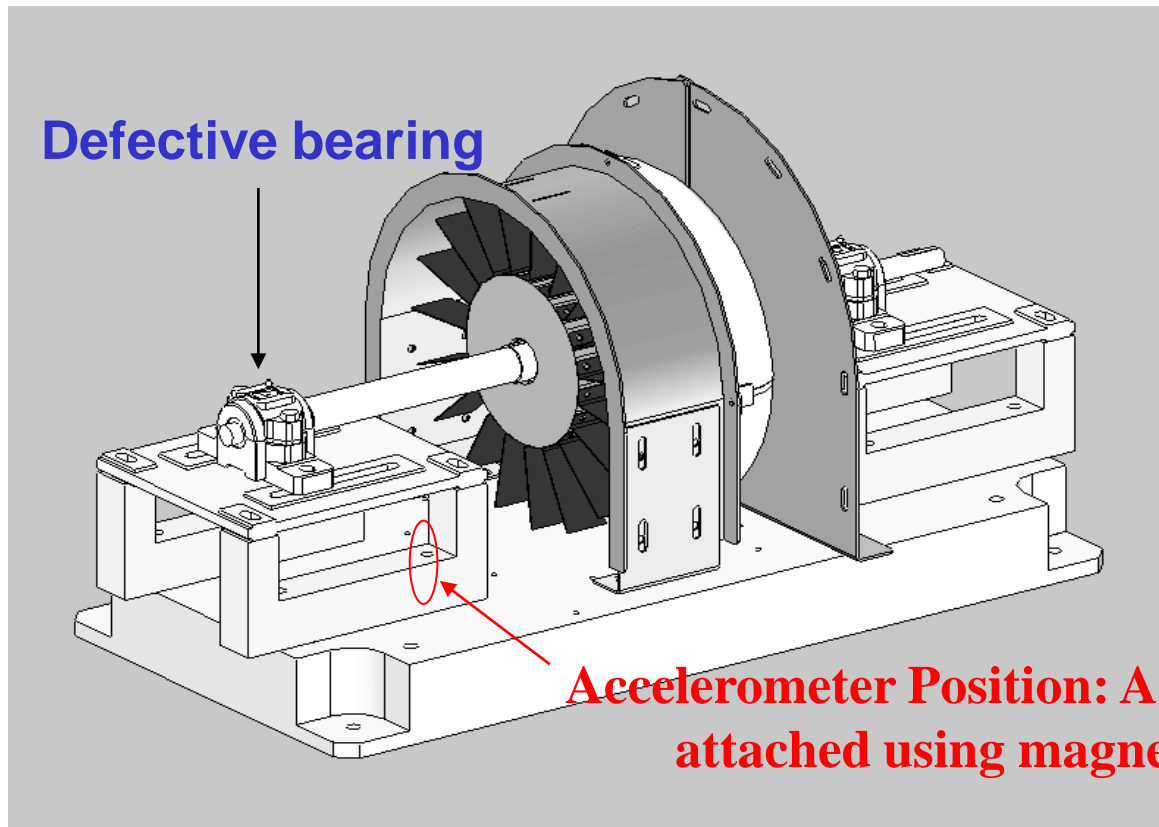
Raw signal

*Residual signal
(after removing
synchronous
average)*

*Residual signal
after editing
the Cepstrum*

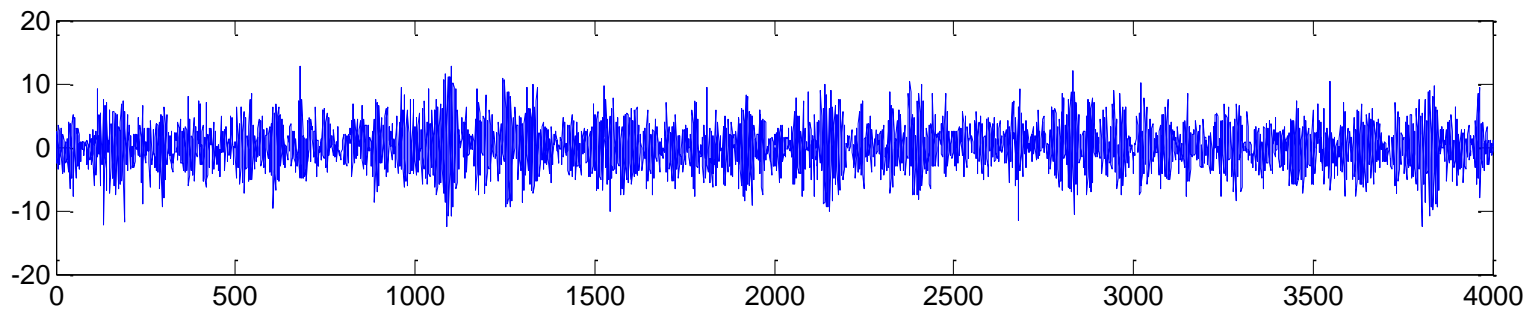


Application to UNSW Fan Test rig Outer Race Fault

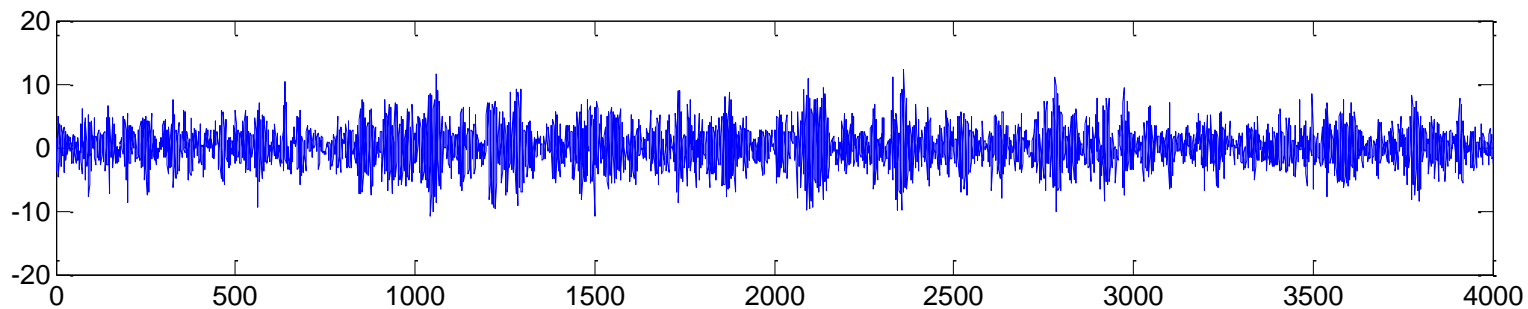




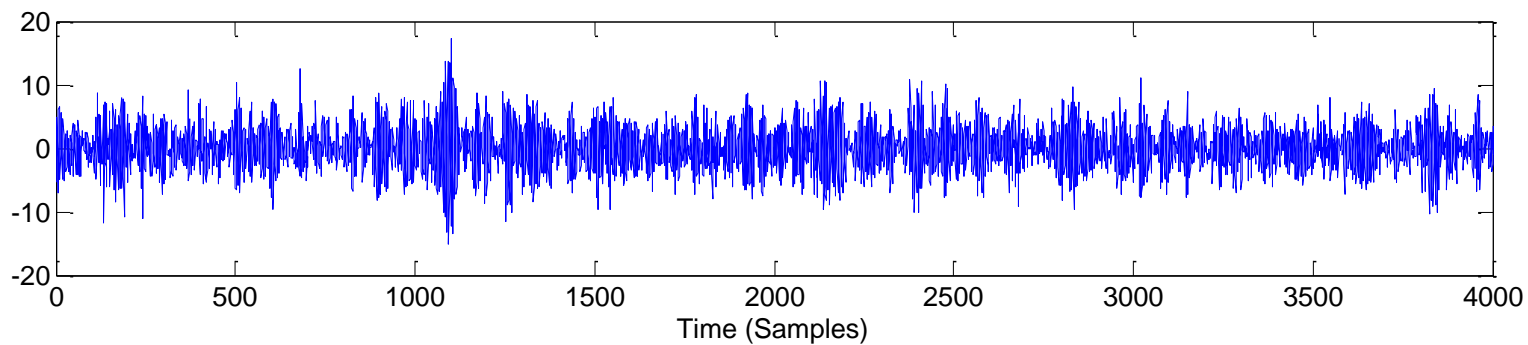
Time Domain Signals



Original



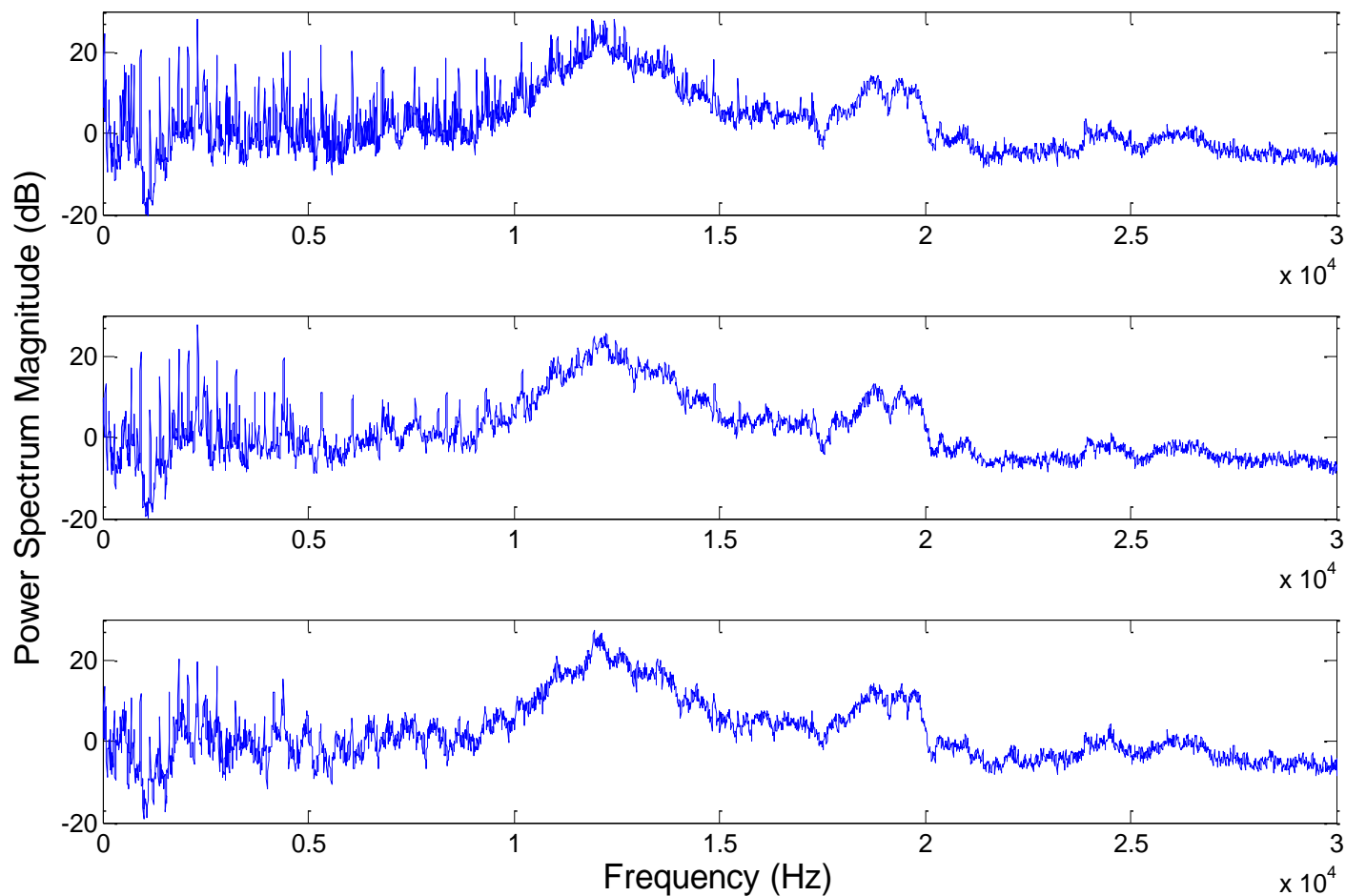
TSA



Cepstrum



Power Spectra (Full Range)



Original

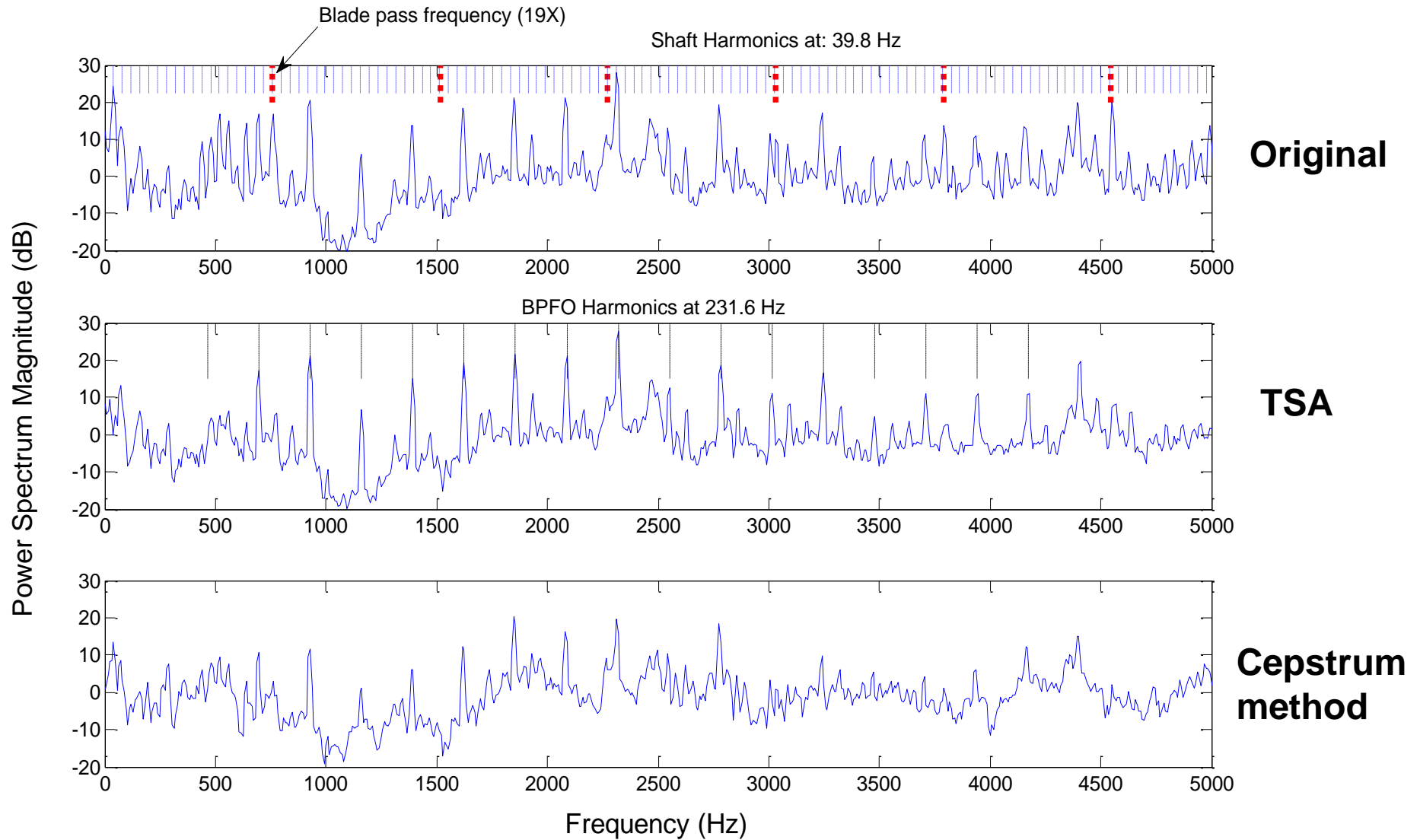
TSA
method

Cepstrum
method

Remaining “periodic” components at low frequency are from bearing fault



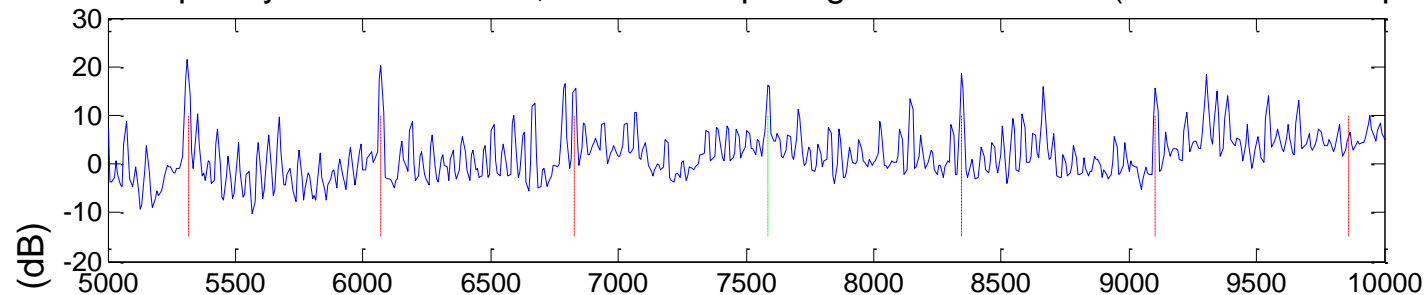
Power Spectra (0-5 kHz)



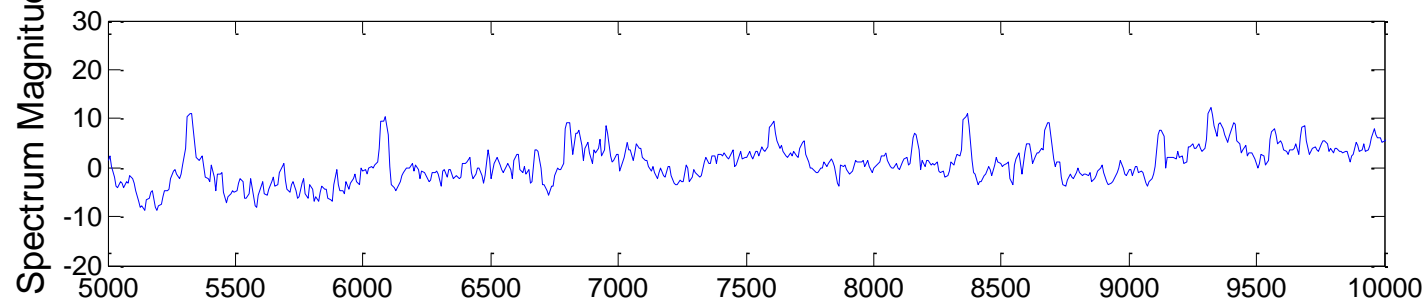


Power Spectra (5-10 kHz)

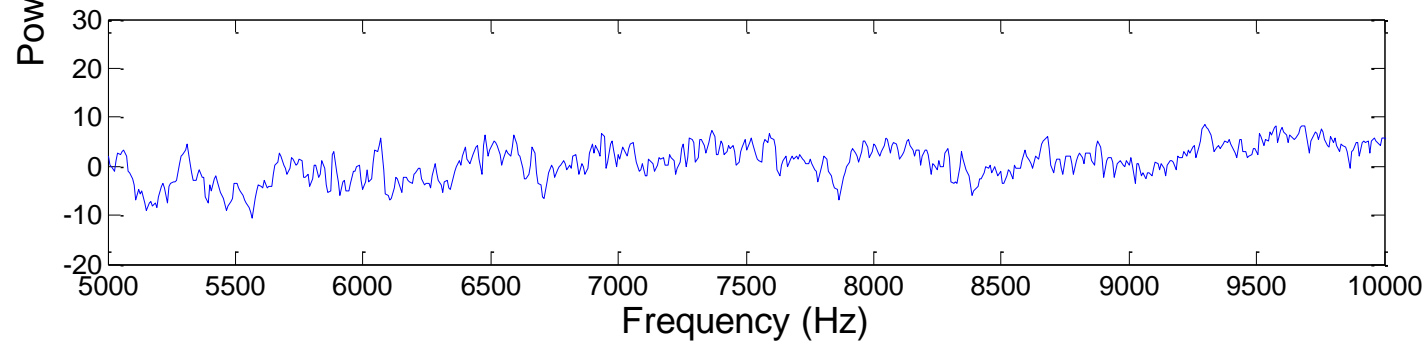
Carrier Frequency at : 7586.41 Hz, Sideband Spacing at : 757.369 Hz (Blade Pass Frequency)



Original



TSA

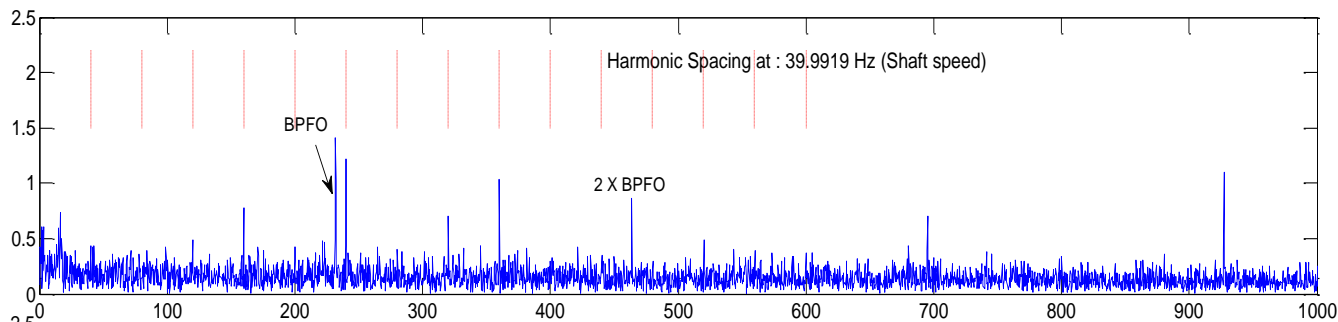


**Cepstrum
method**



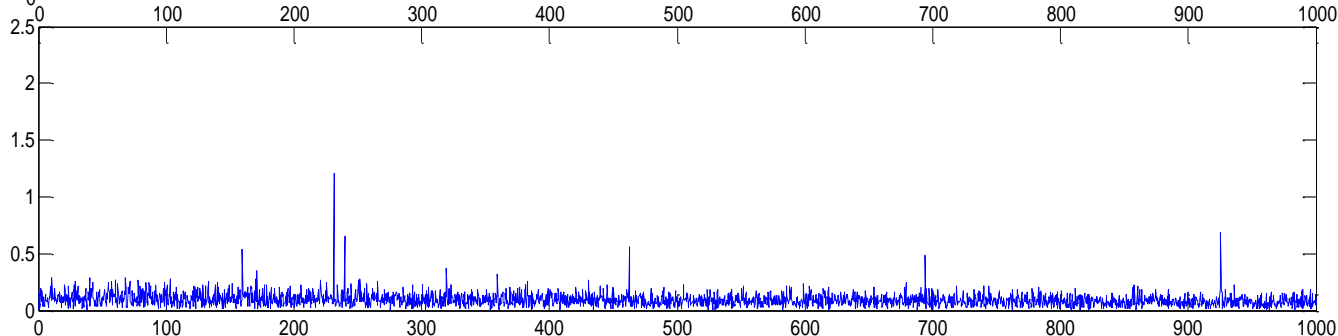
Comparing the envelope spectrum using three methods

Squared envelope spectrum (Bandpass 1000 Hz - 45000 Hz)



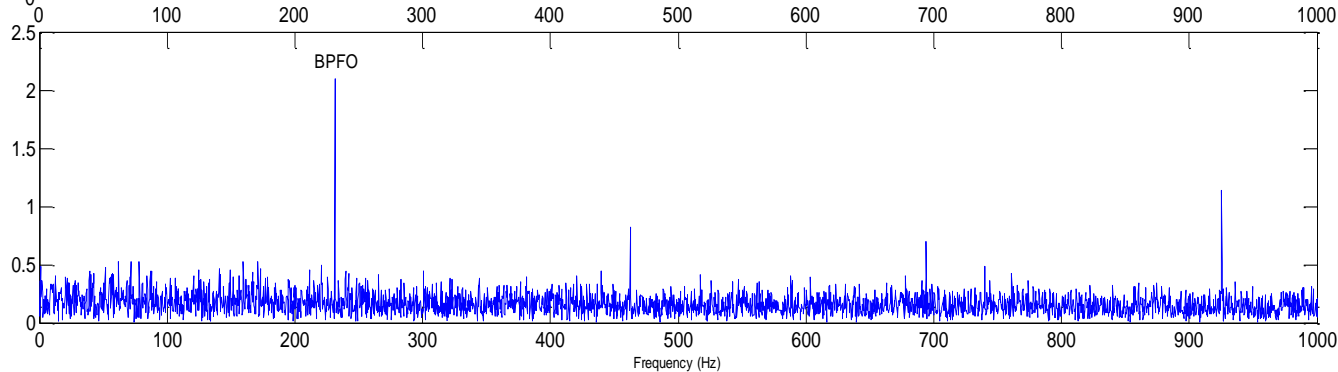
TSA

(a)



DRS

(b)

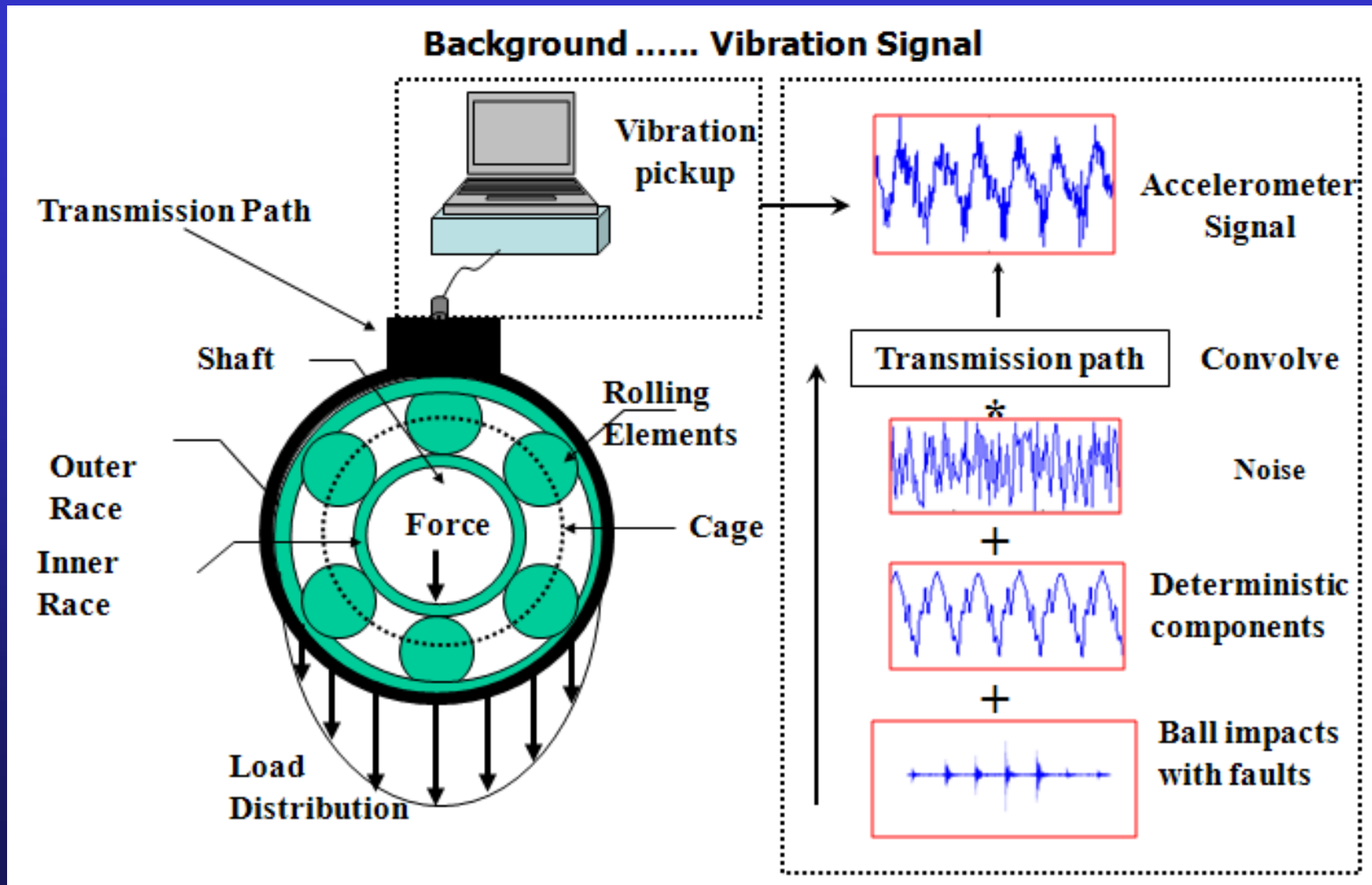


Cepstrum
method

(c)



SEMI-AUTOMATED METHOD for Bearing Diagnostics



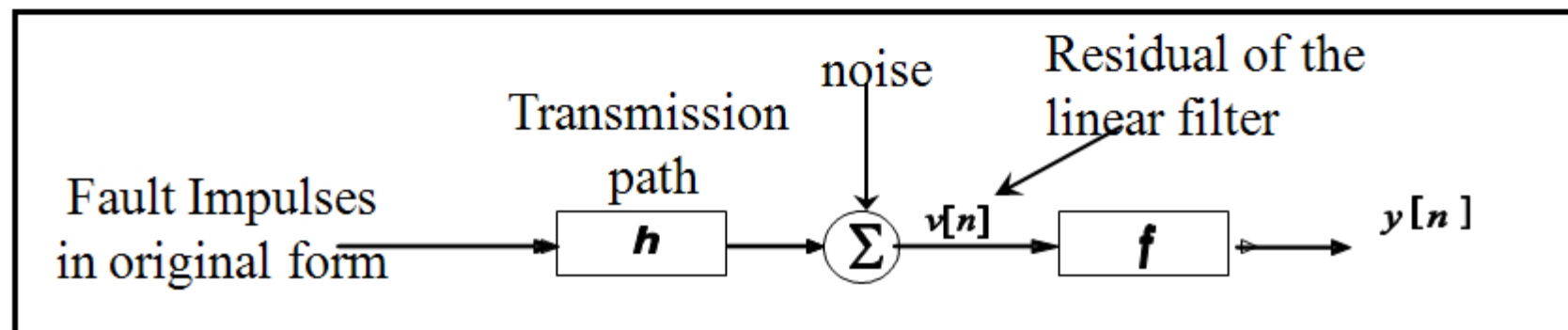


Semi-Automated Bearing Analysis Procedure

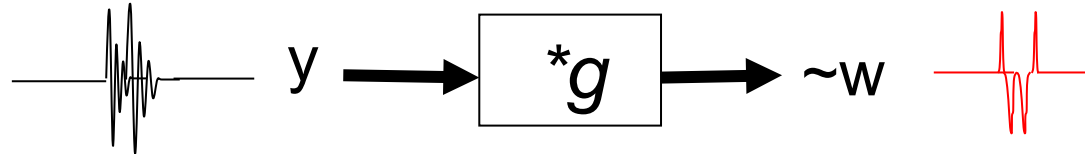
- 1** Order tracking – Remove speed fluctuation
- 2** DRS, SANC or Linear Prediction - Remove discrete frequencies
- 3** MED – Remove smearing effect of signal transfer path
- 4** SK – Determine optimum band for filtering and demodulation
- 5** Envelope analysis – Determine fault characteristic frequencies

The MED Technique

- The MED technique effectively deconvolves the effect of the transmission path and clarifies the impulses.
- It was originally developed by (Wiggins, 1978) to aid extraction of reflectivity information in seismic data.
- It has been recently used by (Endo and Randall, 2004) to enhance impulses arising from faults (Spalls and cracks) in gears.
- The MED searches for an optimum set of filter coefficients that recover the output signal (of an inverse filter f) with the maximum value of kurtosis.



MINIMUM ENTROPY DECONVOLUTION (MED)



Wiggins' Minimum Entropy Deconvolution:

$$w(i) = \sum_{l=1}^L g[l] \cdot y[i-l]$$
$$K(g(l)) = \sum_{i=1}^N w^4(i) / \left[\sum_{i=1}^N w^2(i) \right]^2$$

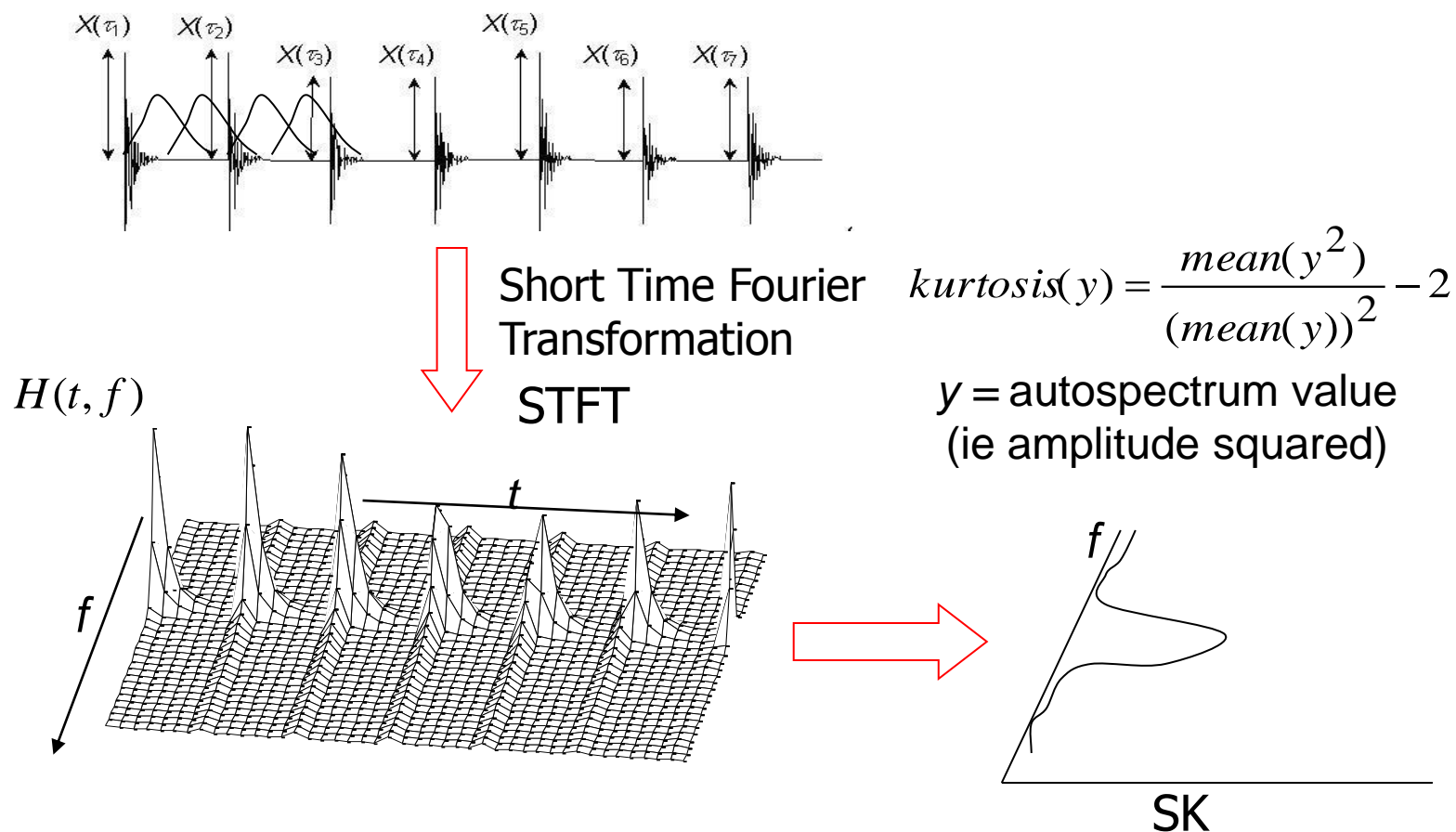
The diagram shows a red bracket under the $w(i)$ equation, with two red arrows pointing from the bracket to the $w^4(i)$ and $w^2(i)$ terms in the $K(g(l))$ equation.

Basic idea is to maximize K by varying $g(l)$:

... solve for minimum value of $\frac{\partial K}{\partial g(l)}$

SPECTRAL KURTOSIS

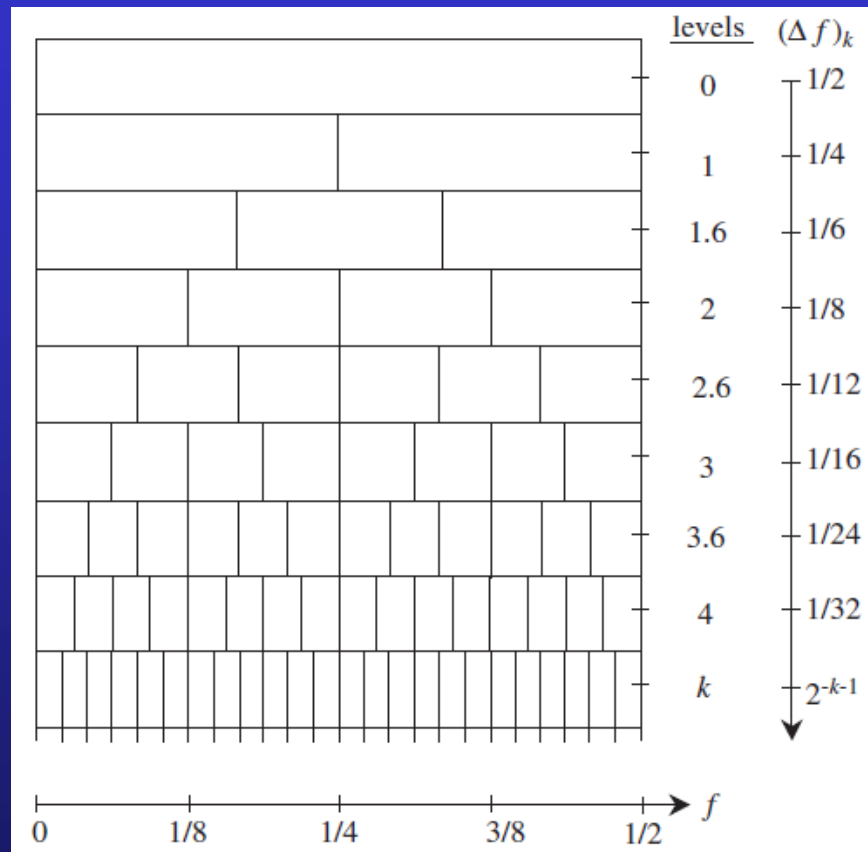
Gives kurtosis (impulsiveness) for each frequency line
in a time-frequency diagram



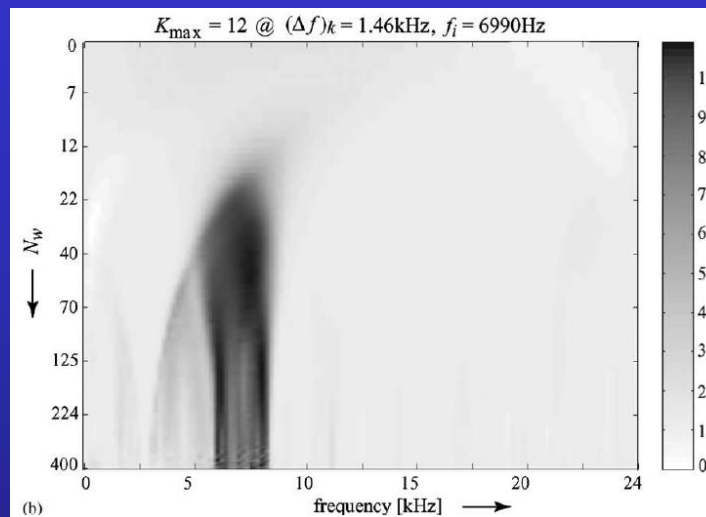


Fast Kurtogram

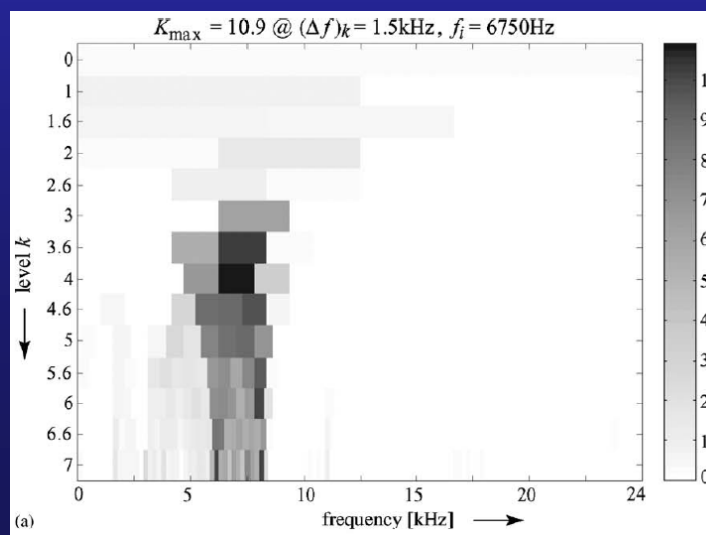
J. Antoni (2006) "Fast computation of the kurtogram for the detection of transient faults", *Mechanical Systems and Signal Processing*, 21(1), pp. 108–124



Filter combinations for 1/3 binary tree



Normal kurtogram



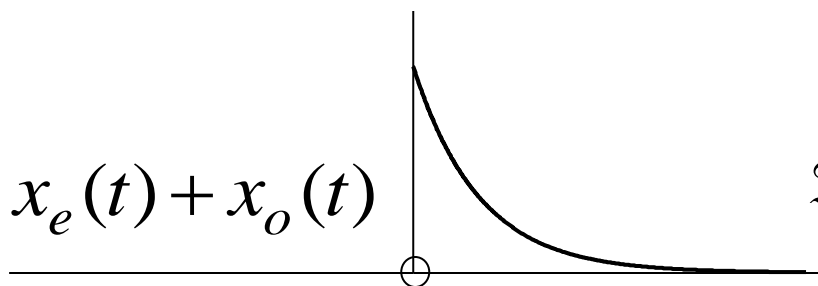
Fast kurtogram



HILBERT TRANSFORM

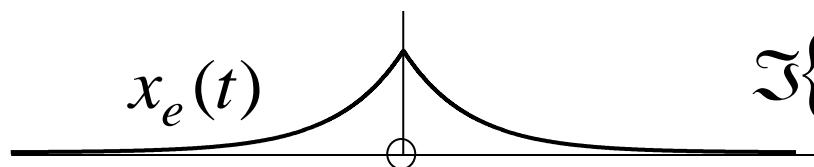
Relationship between the real and imaginary parts
of the Fourier transform of a one-sided function

$$x(t) = x_e(t) + x_o(t)$$



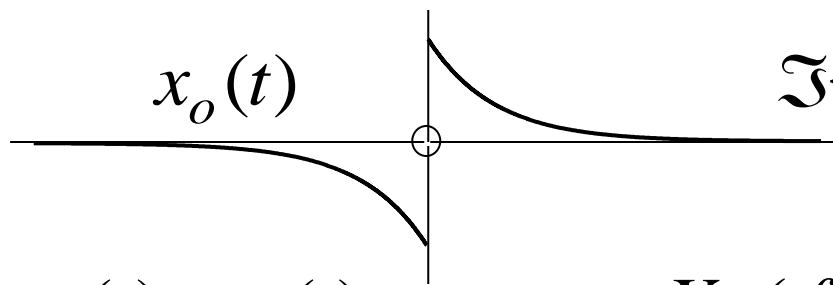
$$\mathfrak{F}\{x(t)\} = X(f)$$

$$x_e(t)$$



$$\mathfrak{F}\{x_e(t)\} = \text{Re}[X(f)]$$

$$x_o(t)$$



$$\mathfrak{F}\{x_o(t)\} = \text{Im}[X(f)]$$

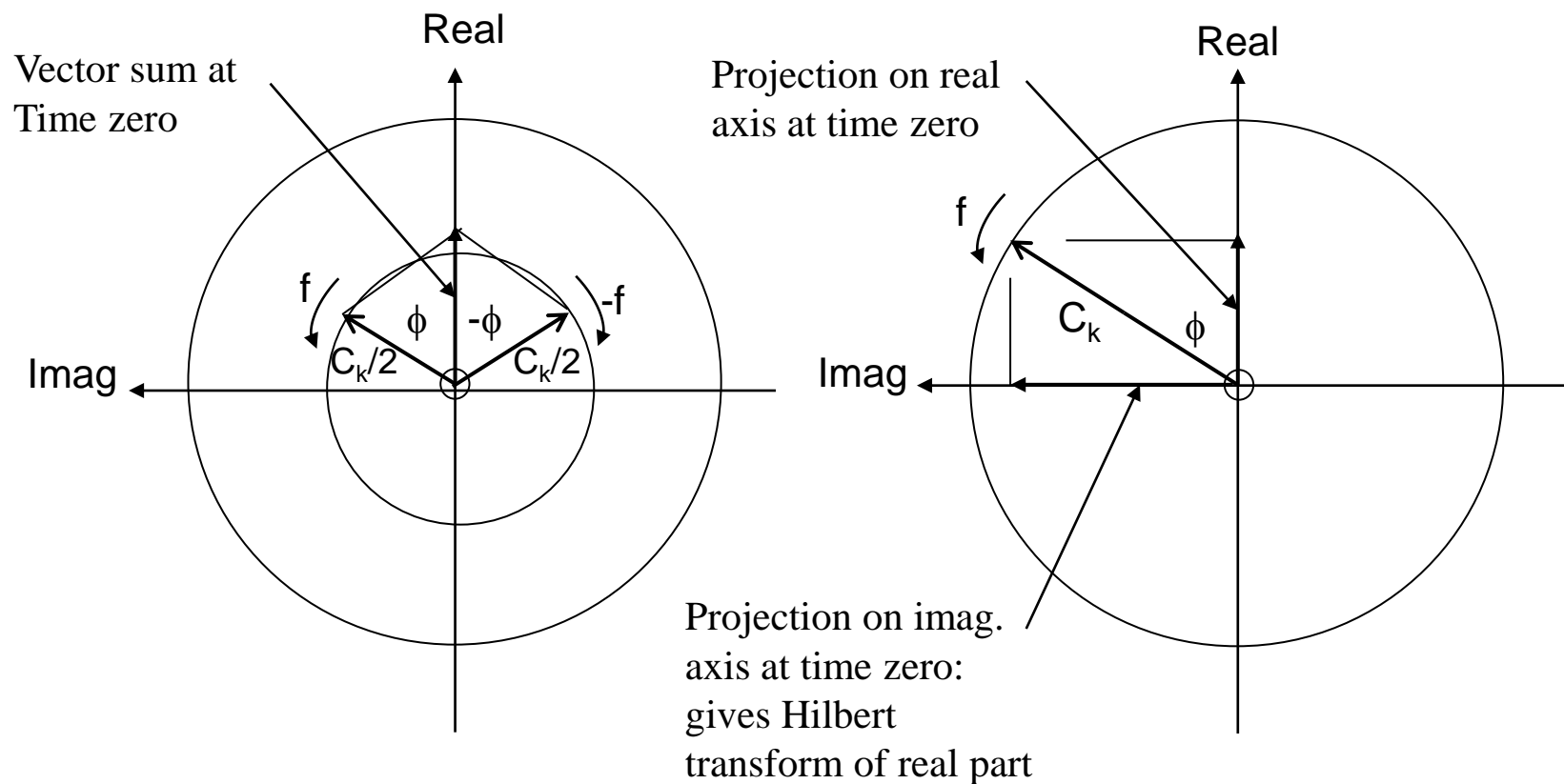
$$x_o(t) = x_e(t) \cdot \text{sgn}(t)$$

$$X_I(f) = X_R(f) * \mathfrak{F}\{\text{sgn}(t)\}$$

ANALYTIC SIGNAL

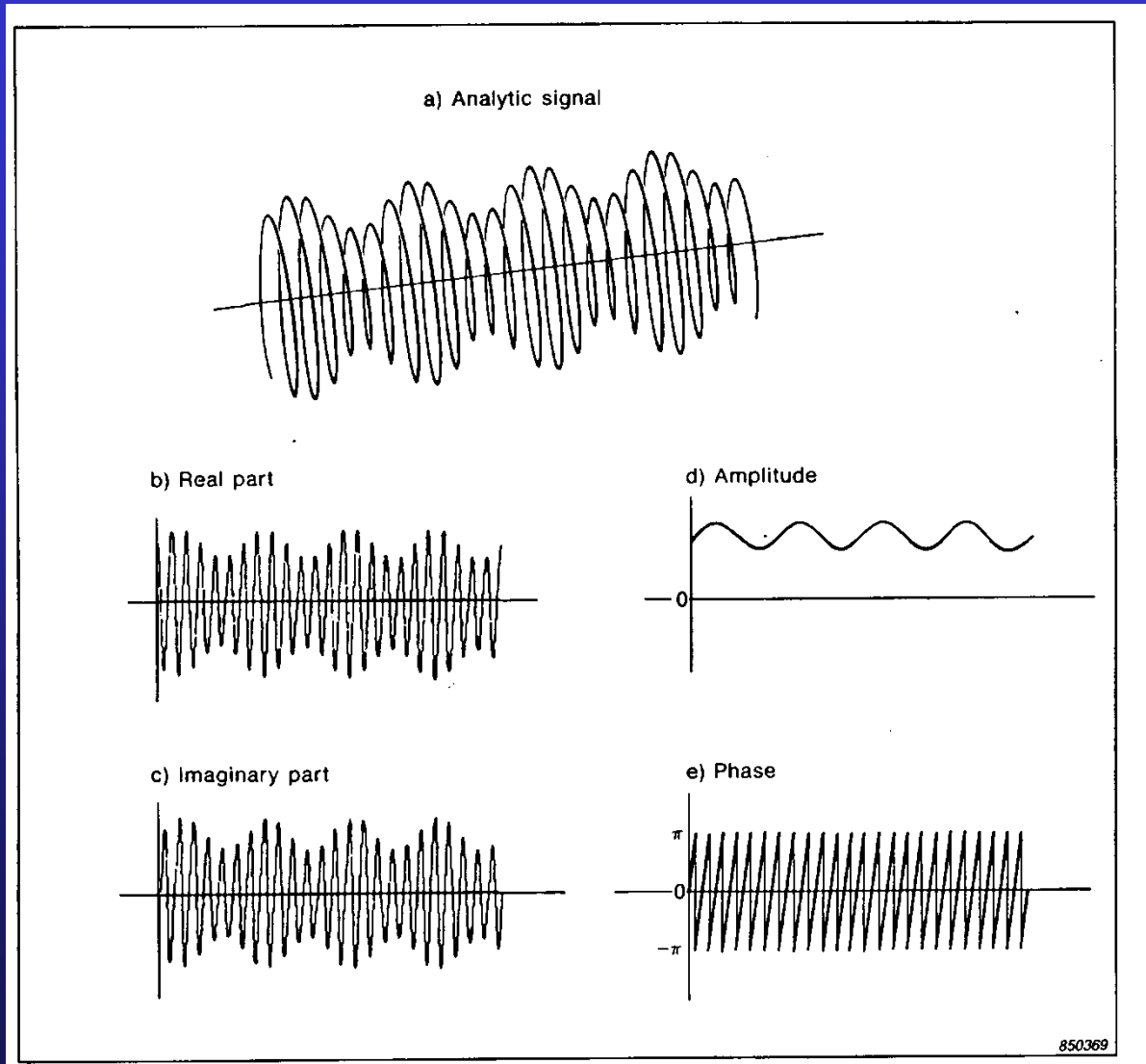
Complex time signal with one-sided spectrum

Real and imaginary parts related by a Hilbert transform



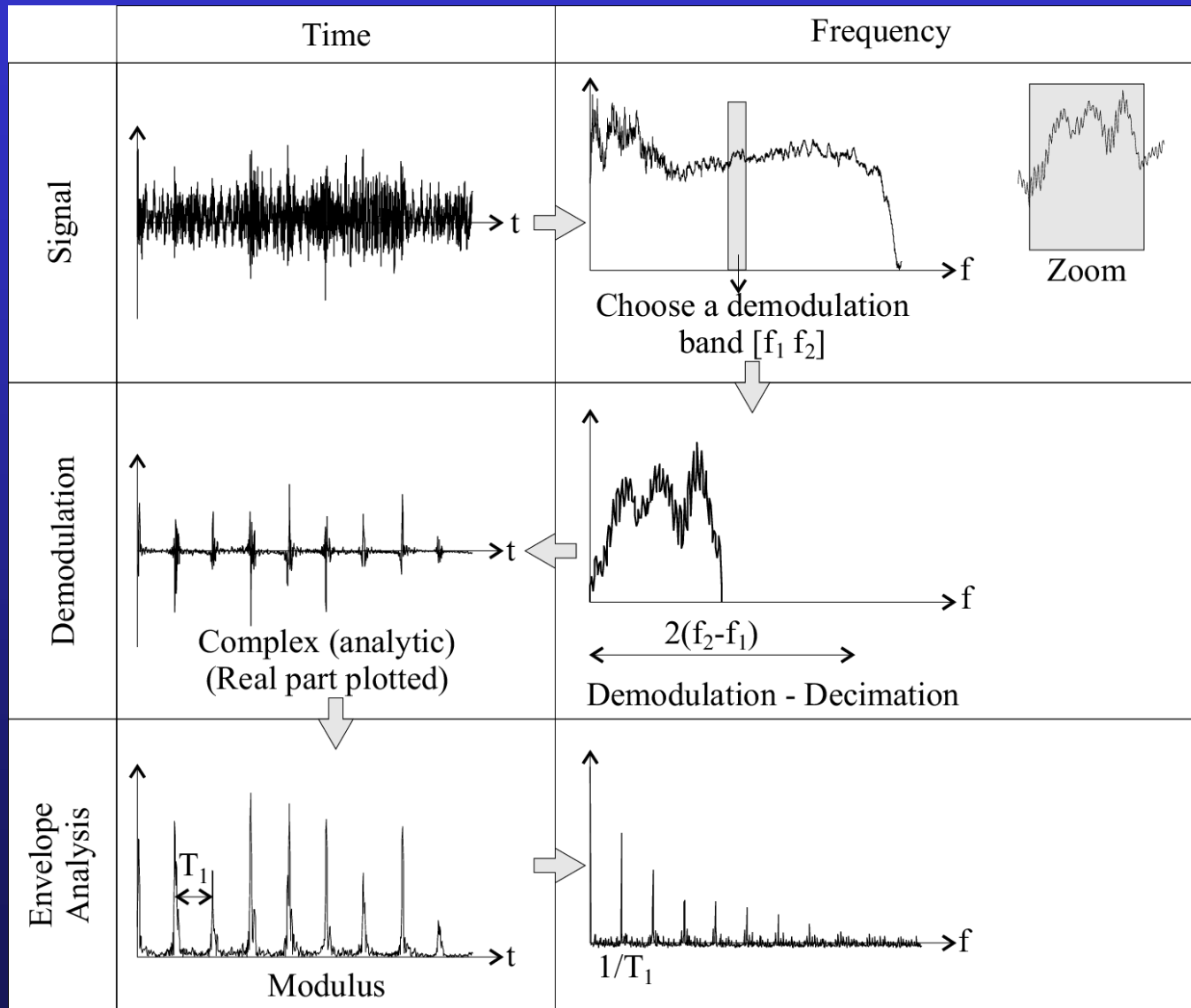


AMPLITUDE MODULATION





HILBERT TECHNIQUE FOR ENVELOPE ANALYSIS



Note that the ideal bandpass filter removes adjacent discrete peaks

Note that 1- sided spectrum values must be complex

It is normally better to analyze the squared envelope rather than the envelope

Advantage of using one-sided spectrum

If the analytic signal (from the one-sided spectrum) is termed $f_a(t)$, its squared envelope is formed by multiplication with its complex conjugate, and the spectrum of the squared envelope will be the convolution of the respective spectra. Thus:

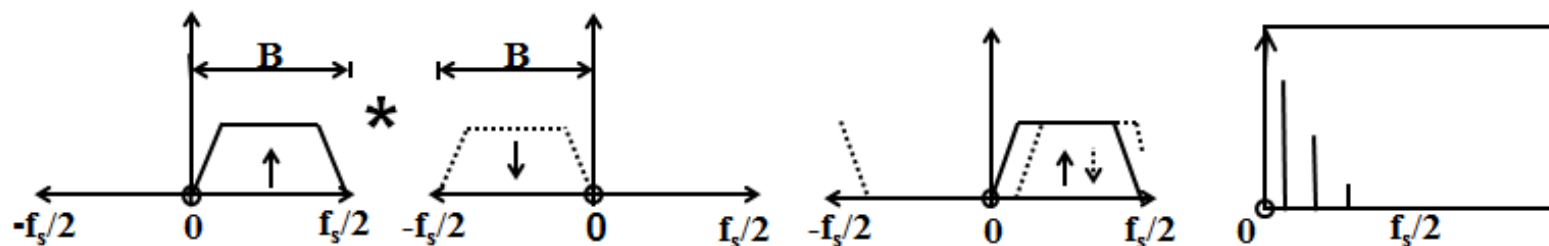
$$\mathfrak{T}\{f_a(t) \cdot f_a^*(t)\} = \mathfrak{T}\{f_a(t)\} * \mathfrak{T}\{f_a^*(t)\} = F_a(f) * F_a^*(-f)$$

Spectrum

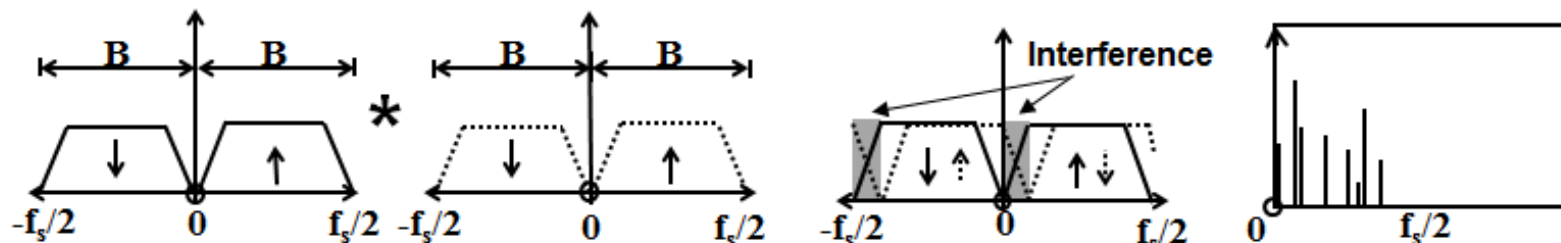
Spectrum

Convolution

**Envelope
Spectrum**



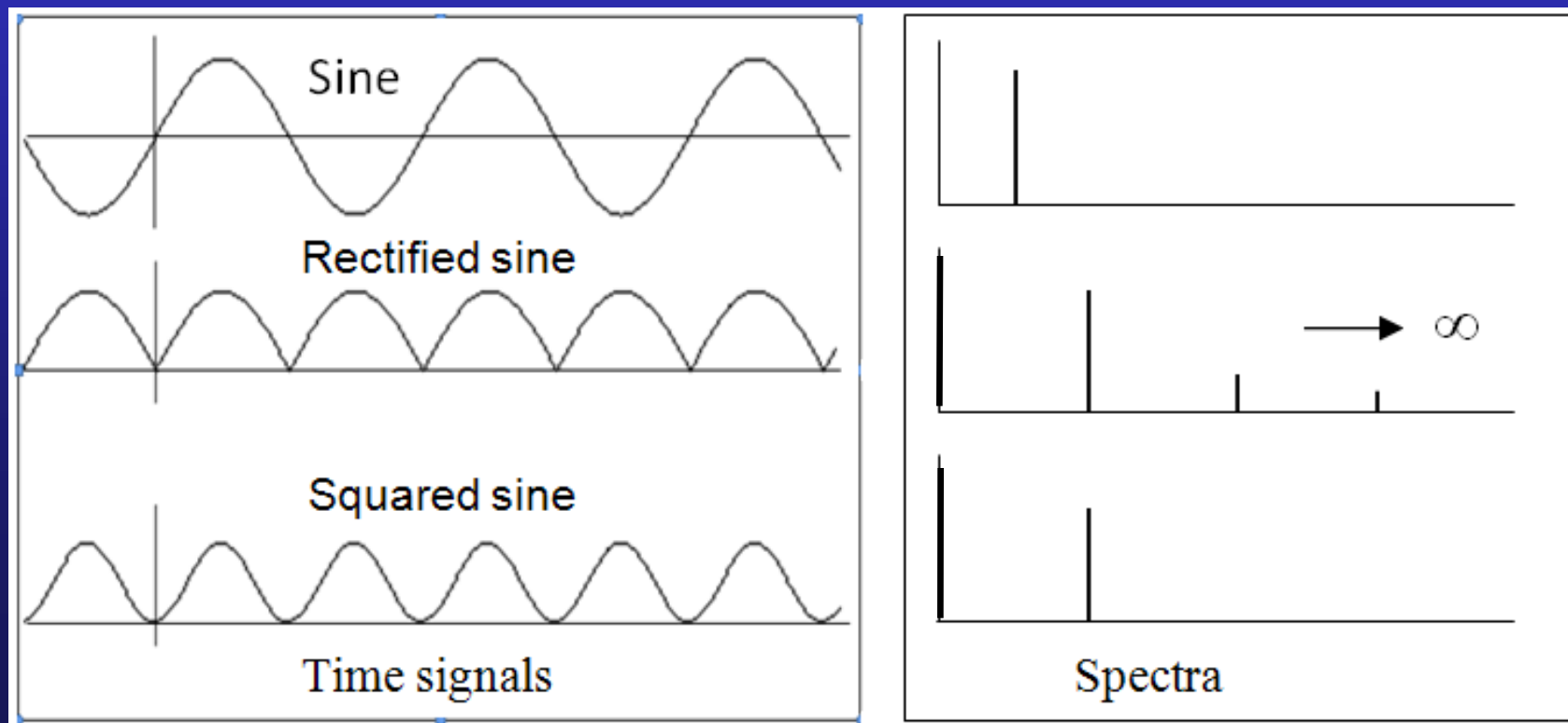
Analytic signal –
difference
frequencies only



Real signal –
also sum
frequencies

Advantages of Squared rather than Rectified Envelope

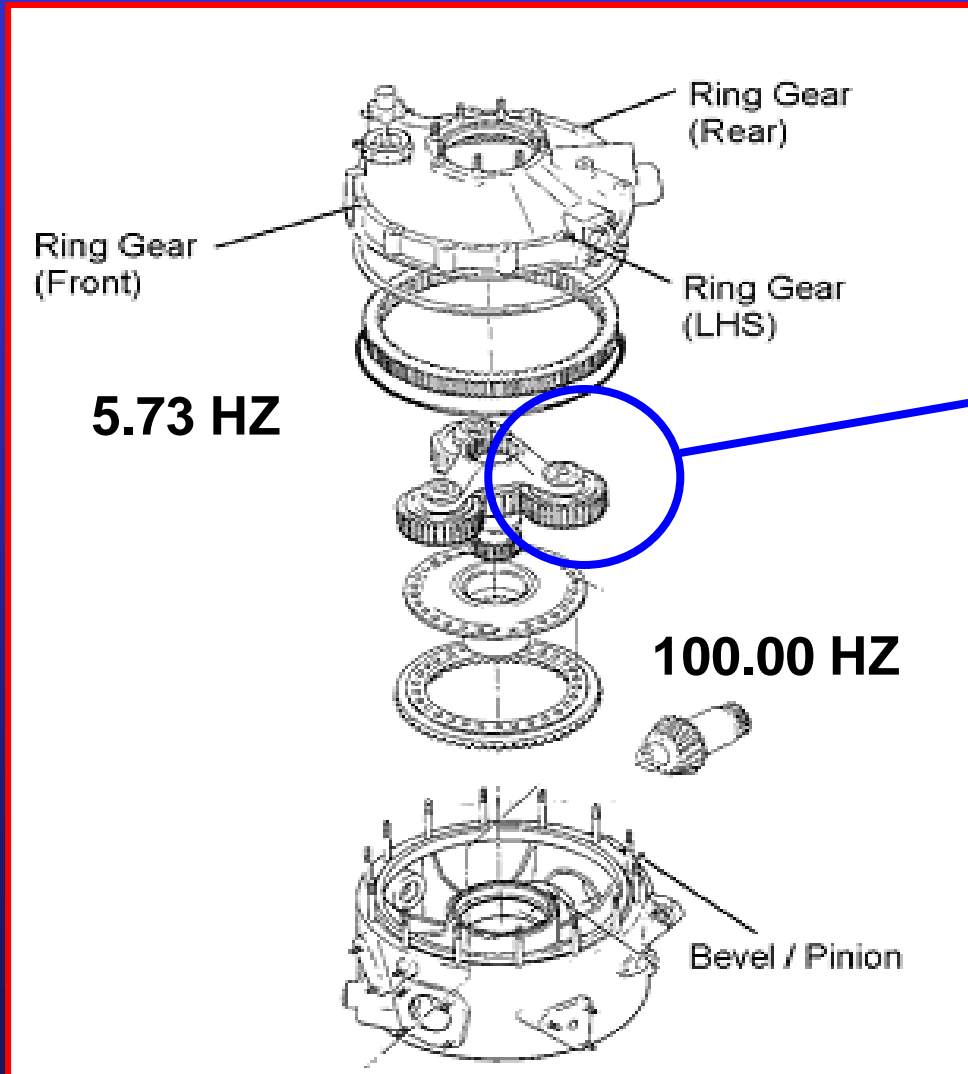
Squared signal contains only DC component plus (double) frequency
Rectified signal has sharp cusps requiring harmonics to infinity which
alias into measurement range (ie avoid taking square root)





Case History – Helicopter Gearbox Rig

Blind analysis

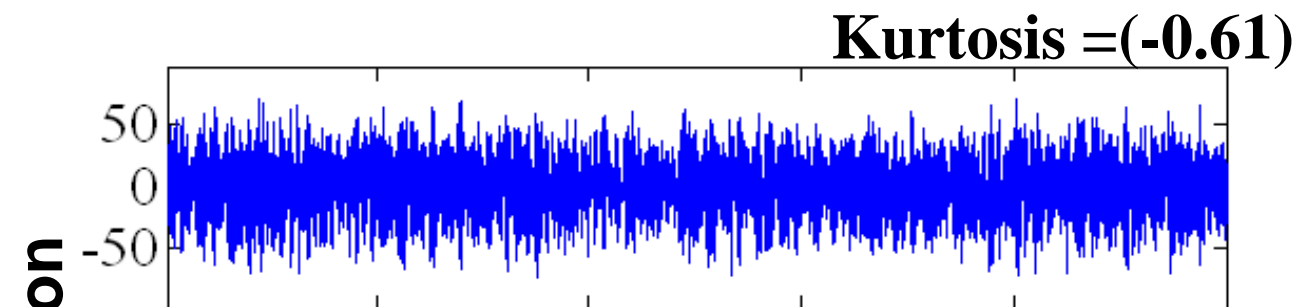


Planetary Bearing

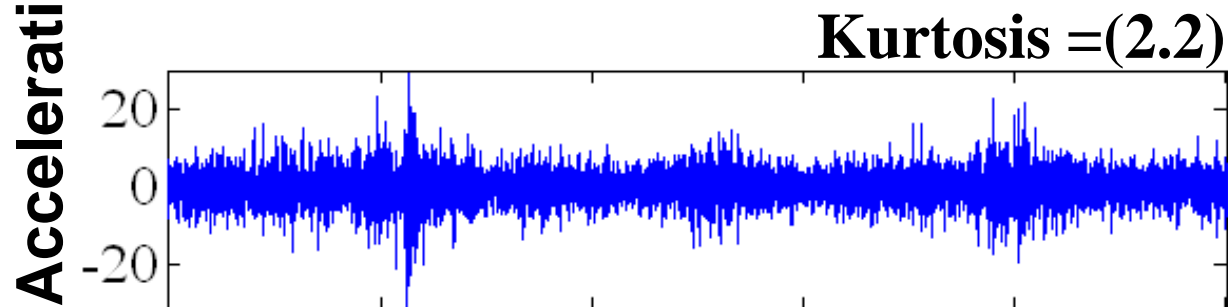


Time domain after filtration

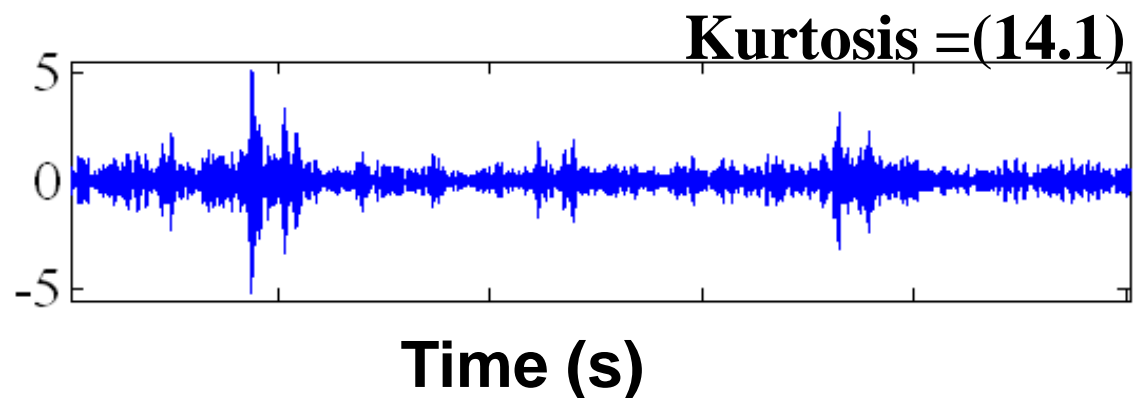
Order tracked signal



Residual signal
after DRS and linear
prediction

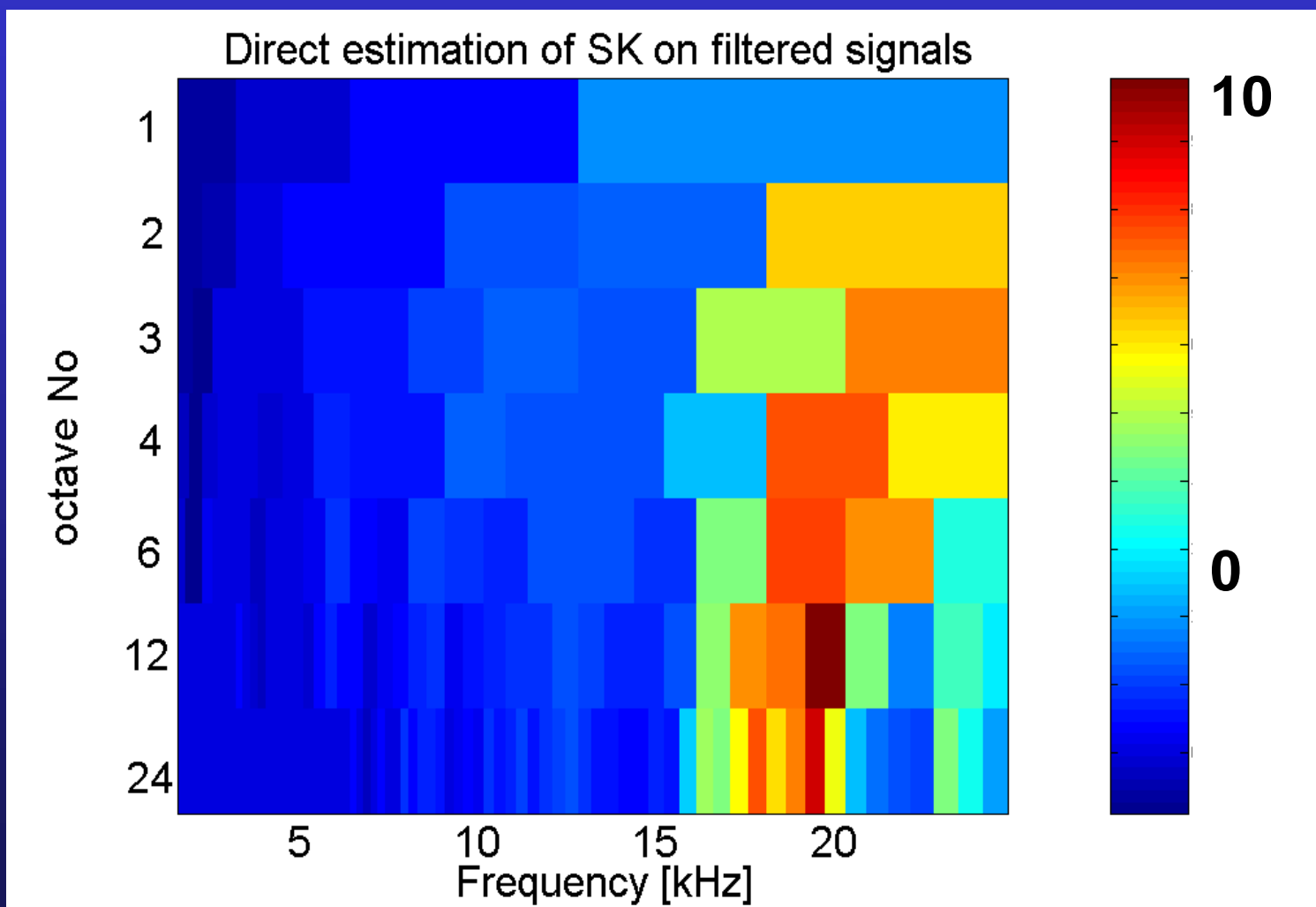


Filtered signal using
SK





SK analysis showing the maximum excited bands





New cepstral pre-whitening technique

- **Based on the new method of editing a time signal by editing the spectrum amplitude in the real cepstrum, then combining with the original phase to return to the time domain**
- **Extreme case is where real cepstrum is set to zero (spectrum amplitude set to one, ie whitened). Both discrete frequencies and resonances removed. Uniform spectrum weighting means that impulsive frequency bands dominate time signals**

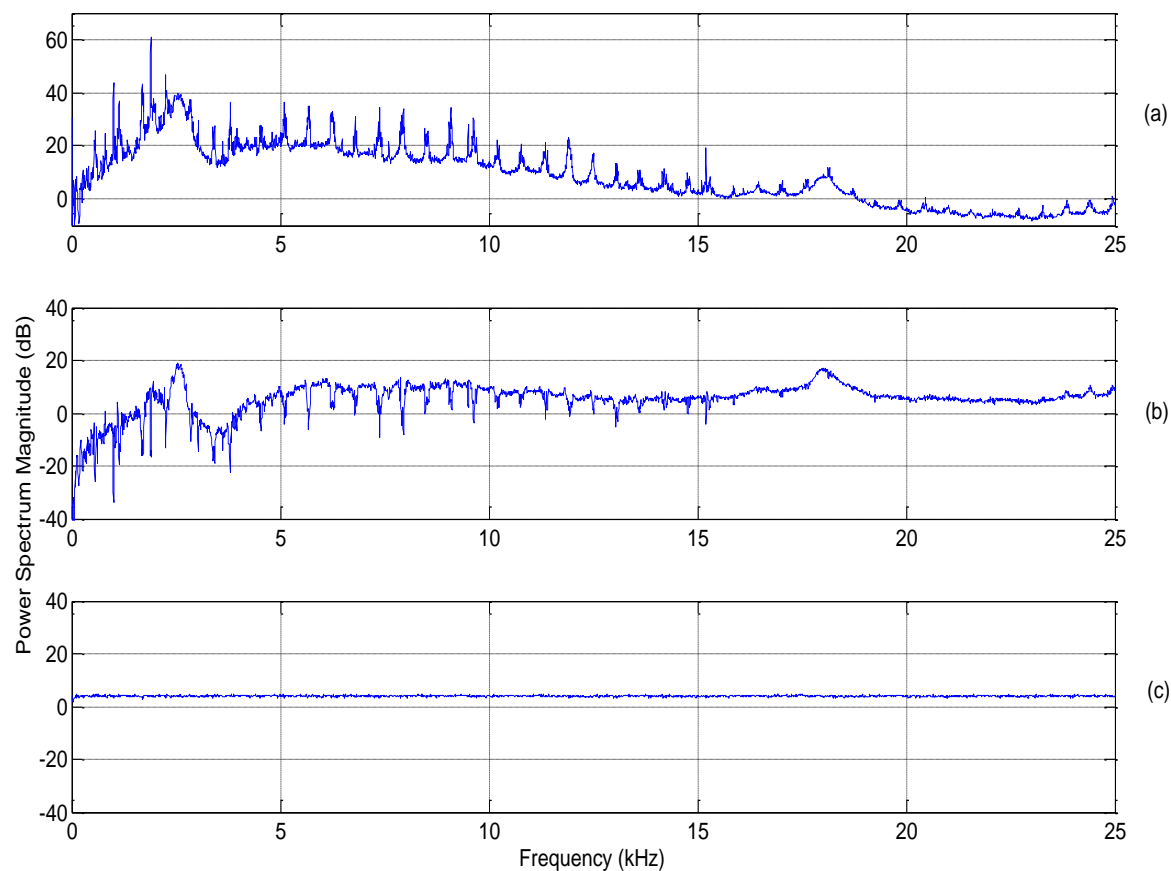


Example of application to the helicopter gearbox signal Spectra

Original spectrum

Whitening using low
order AR model

Cepstral whitening

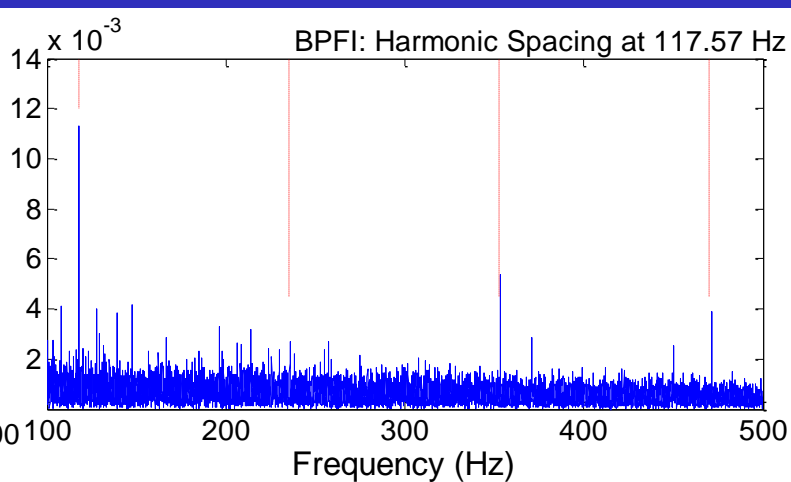
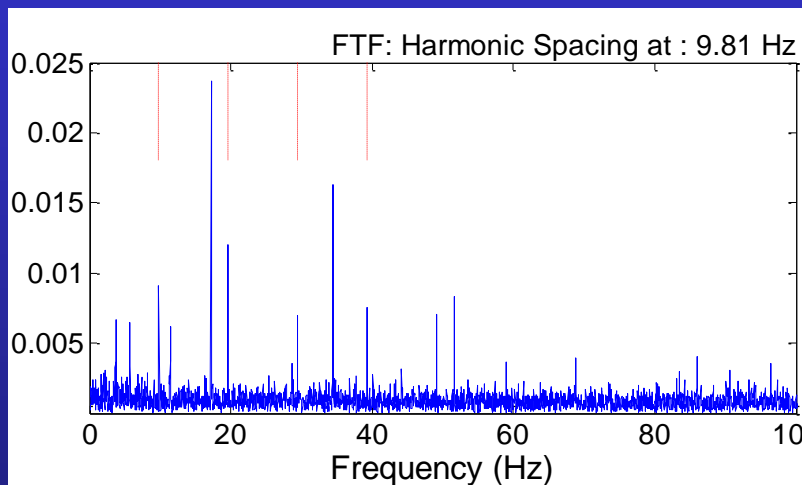




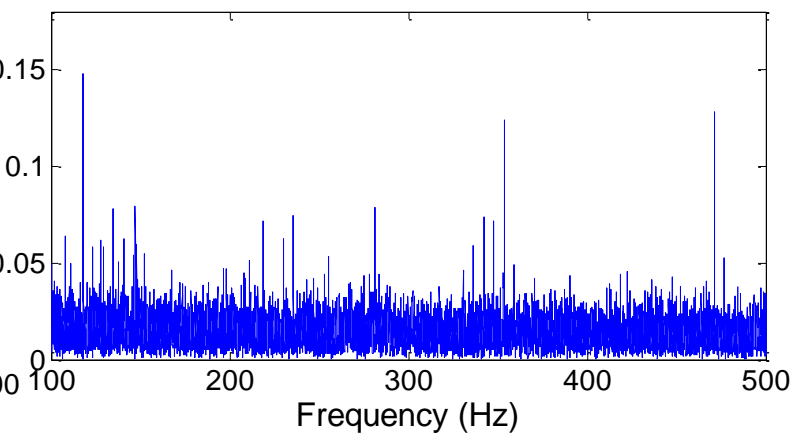
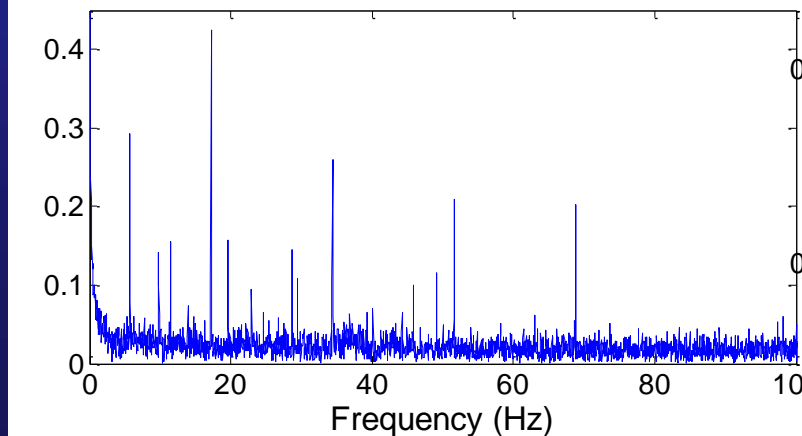
ENVELOPE SPECTRA

Low frequency (FTF)

High frequency (BPFI)



DRS - SK



Cepstrum
whitened



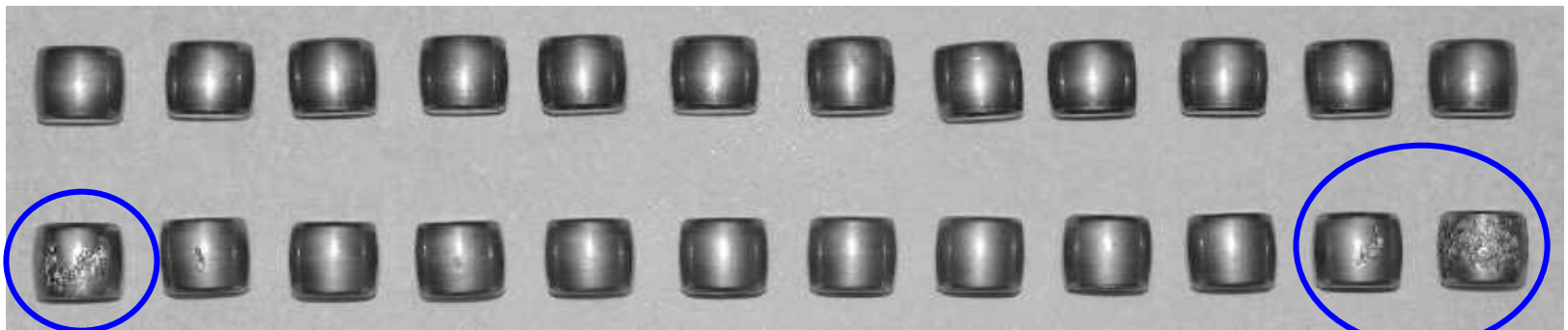
Findings Agree With Analysis Results



Planetary Bearing



Inner Race

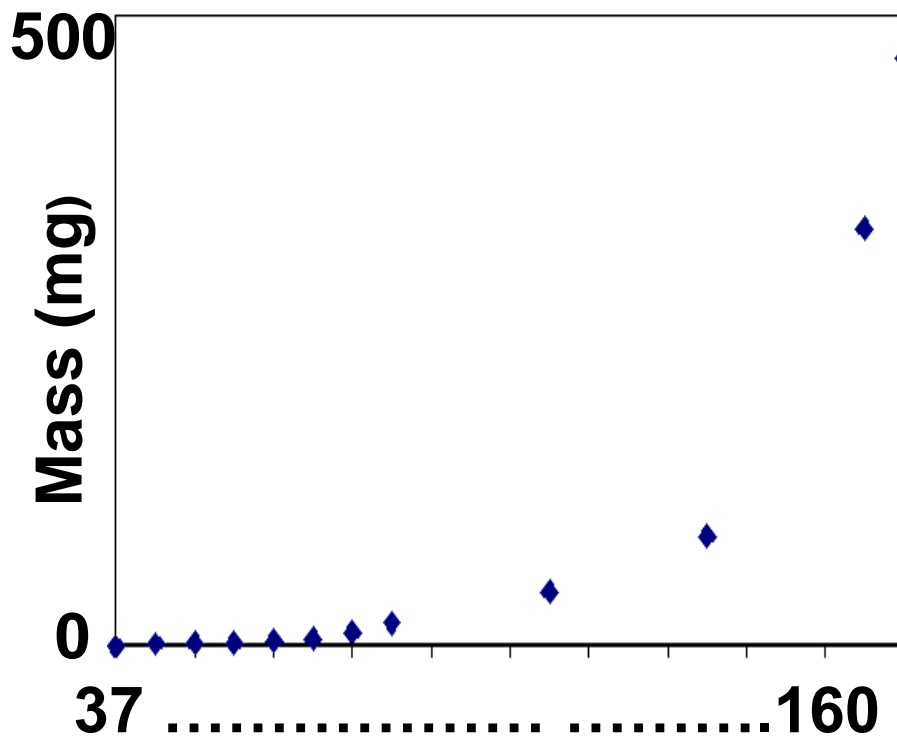


Rollers



Trending based on SK vs. Oil Wear Debris

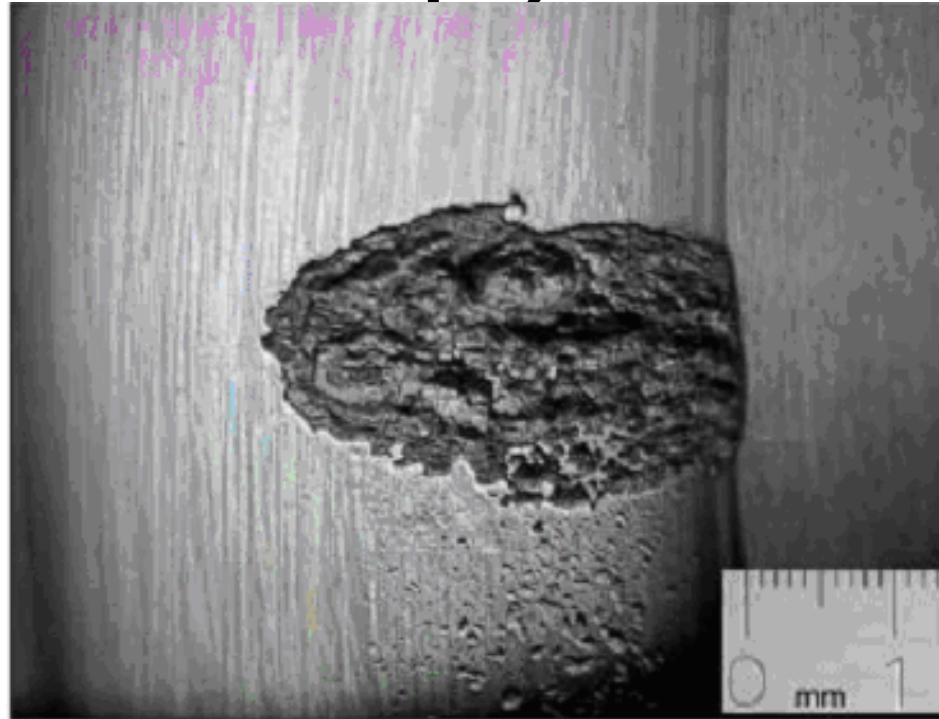
Accumulated oil wear debris





Second Case History High Speed Bearing Test Rig

**FAG Test Rig L17 .. High Speed (12,000
rpm)**



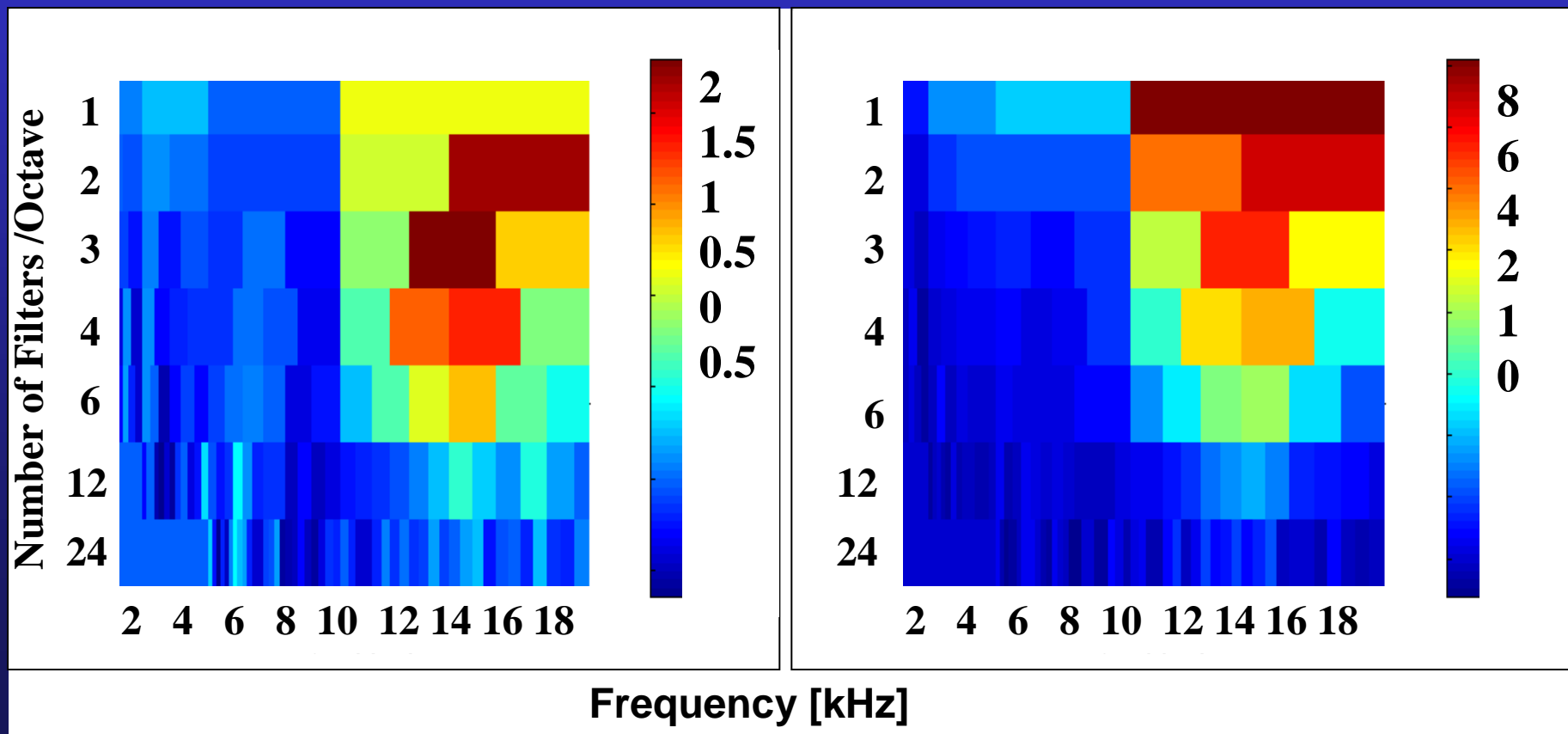
Spall in the inner race



The Effect of using The MED Technique

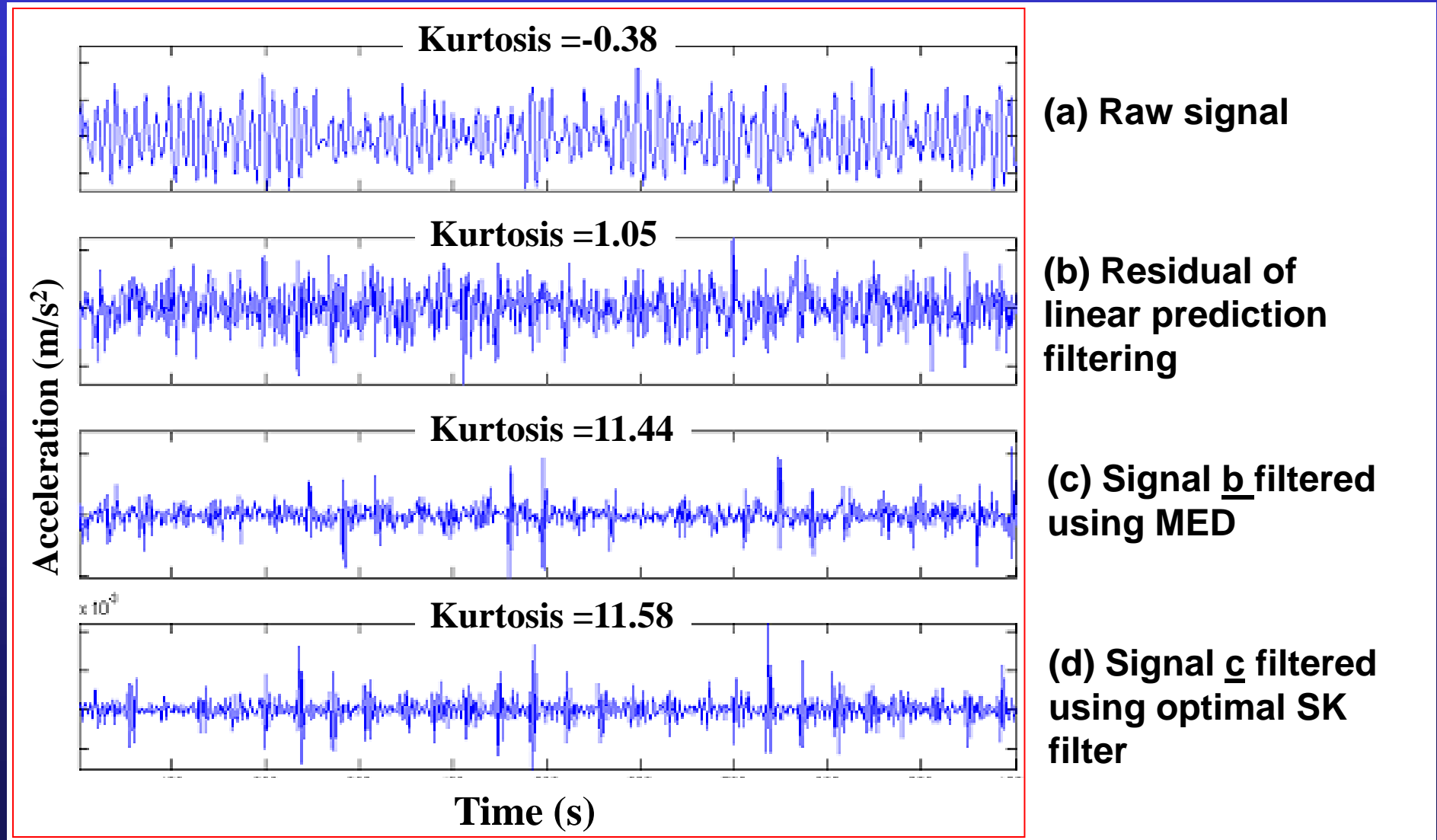
The SK before using the MED

The SK after using the MED



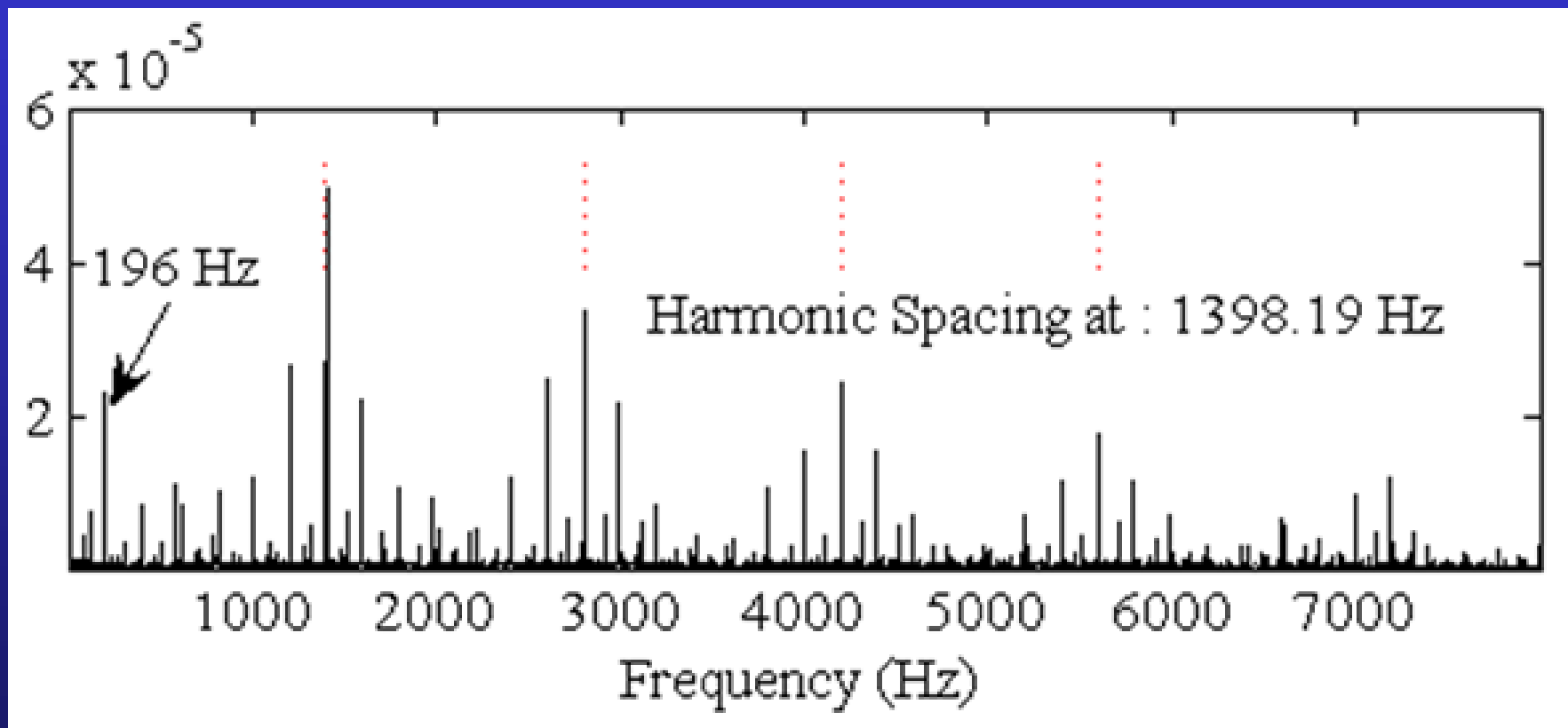


The Effect of using the MED Technique



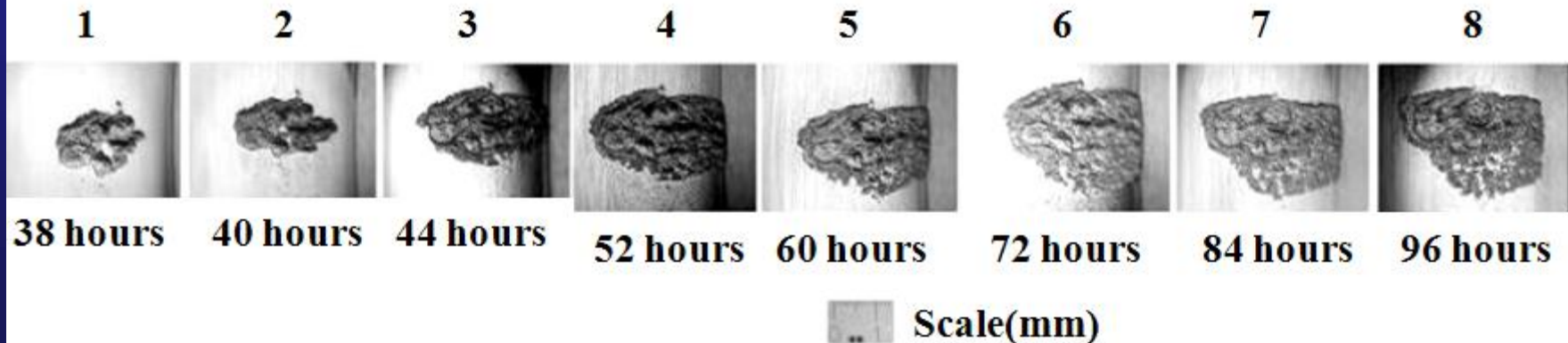
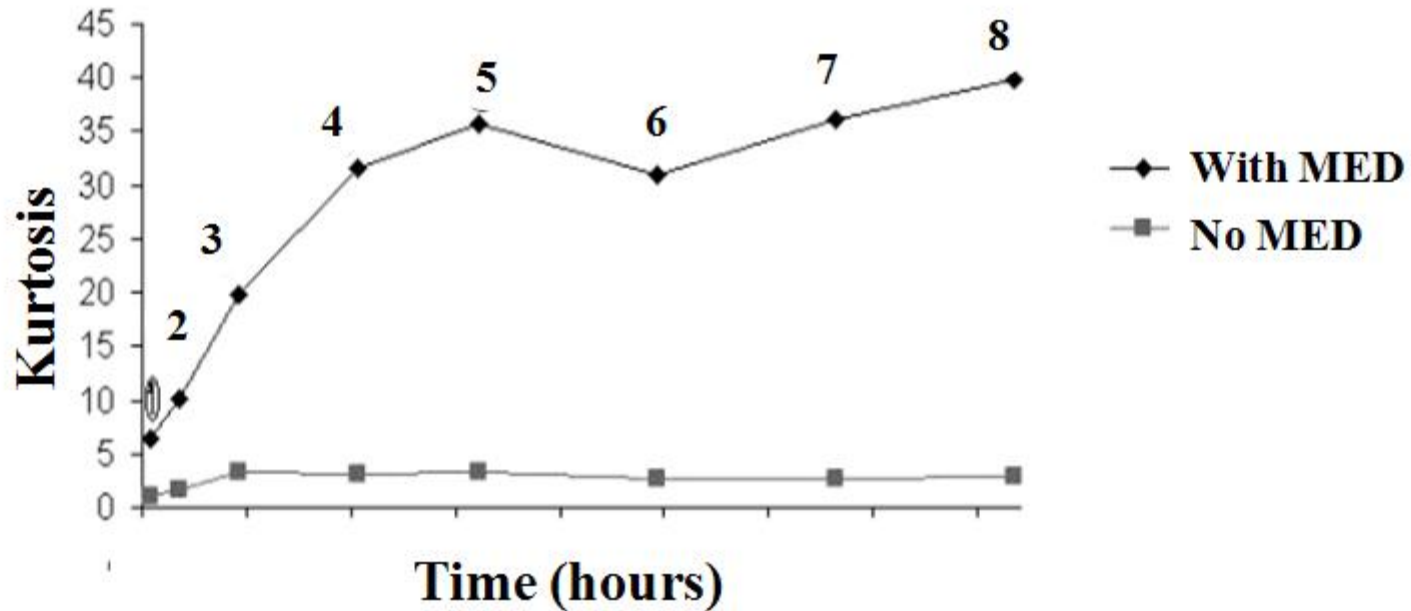


Envelope Analysis after MED and SK

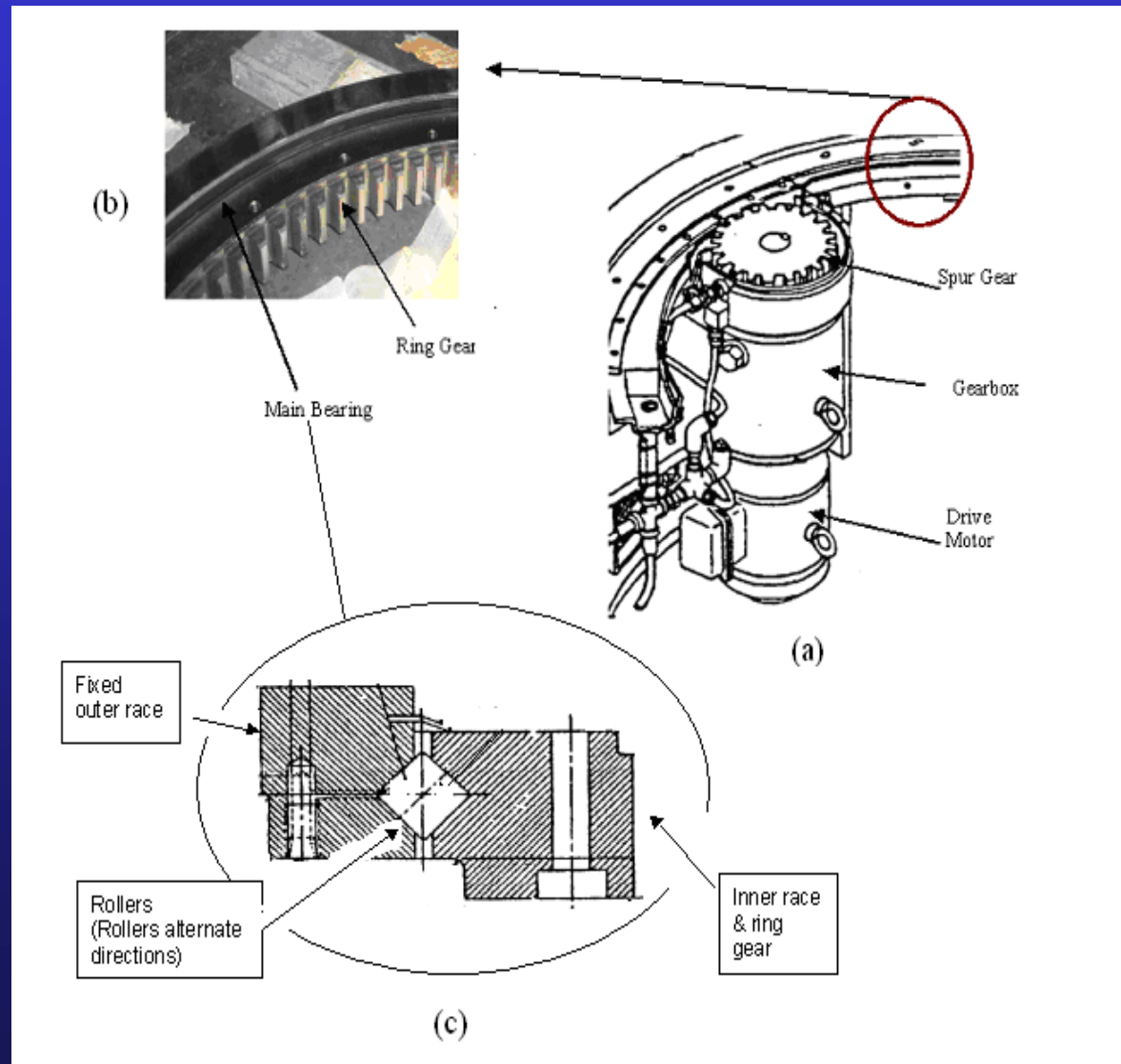


Harmonics at BPF1, sidebands at shaft speed

Trending Fault Development



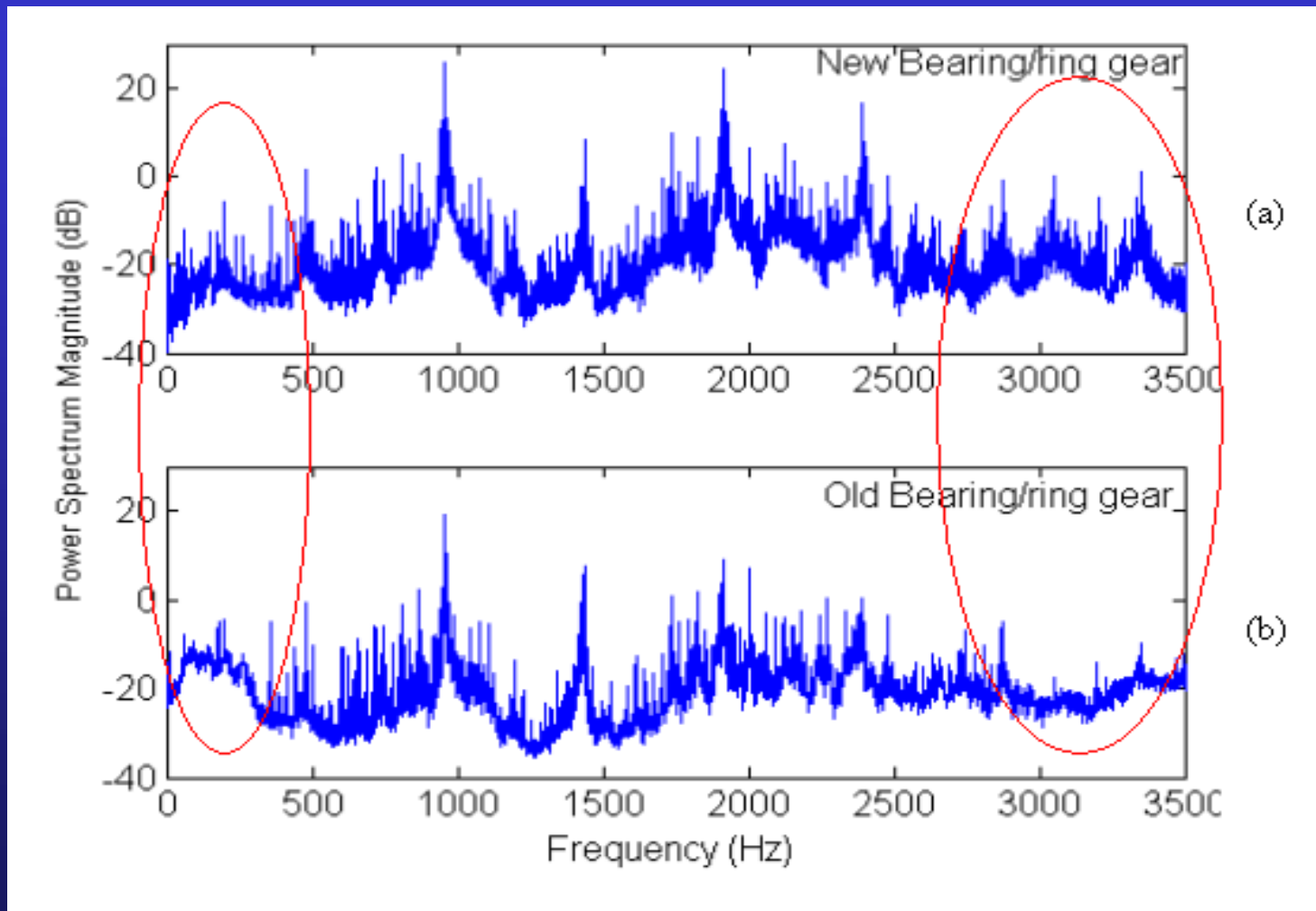
Third Case History – Radar Tower Bearing



- Very slow speed (12 sec period)
- 118 square rollers in alternate directions so each race strikes every second roller
- Bearing and ring gear changed, pinion unchanged



SPECTRUM COMPARISON

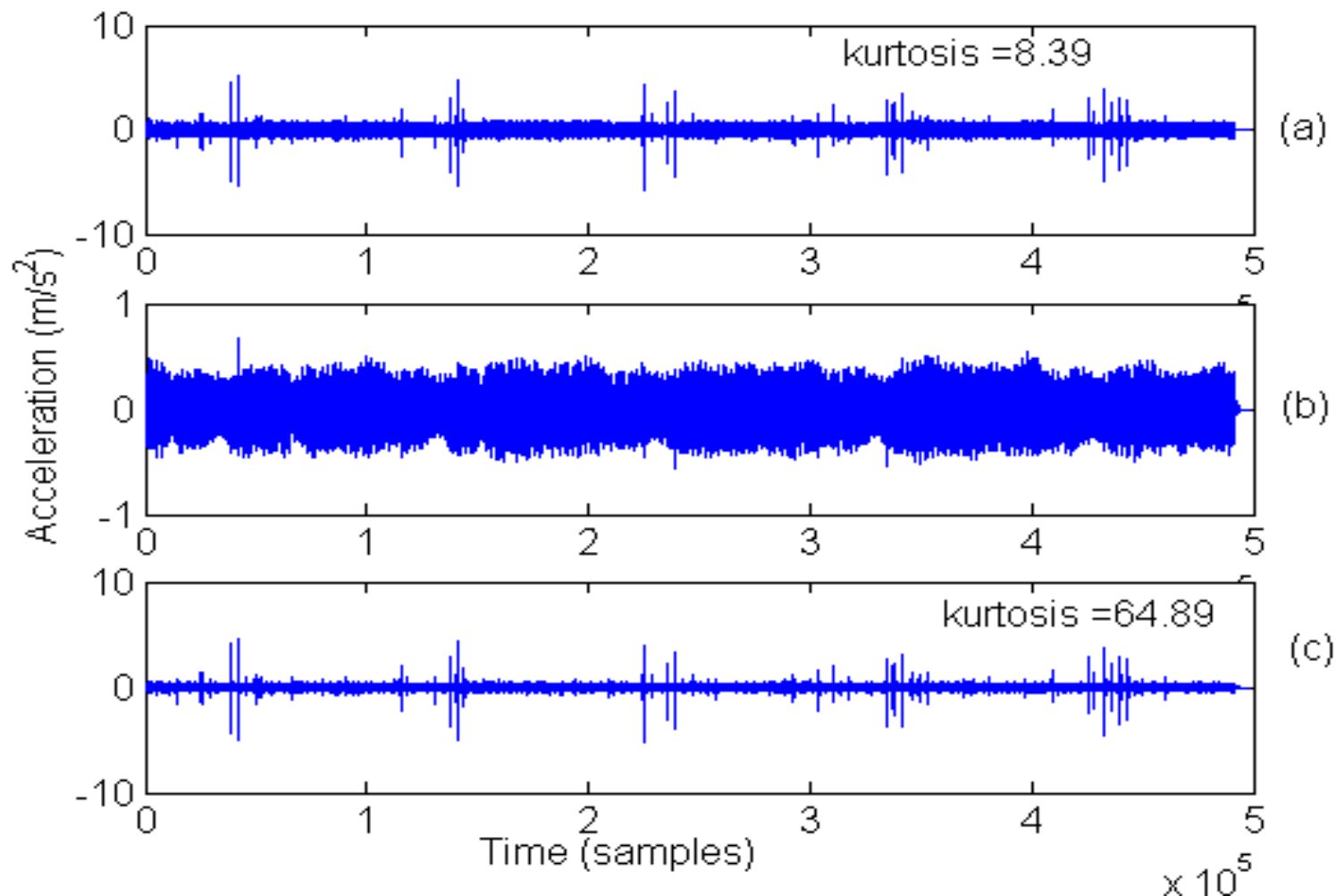


**Dominated by gears, but differences
at high and low frequencies**



Removal of gear signals by DRS

DRS Faulty Bolt 5



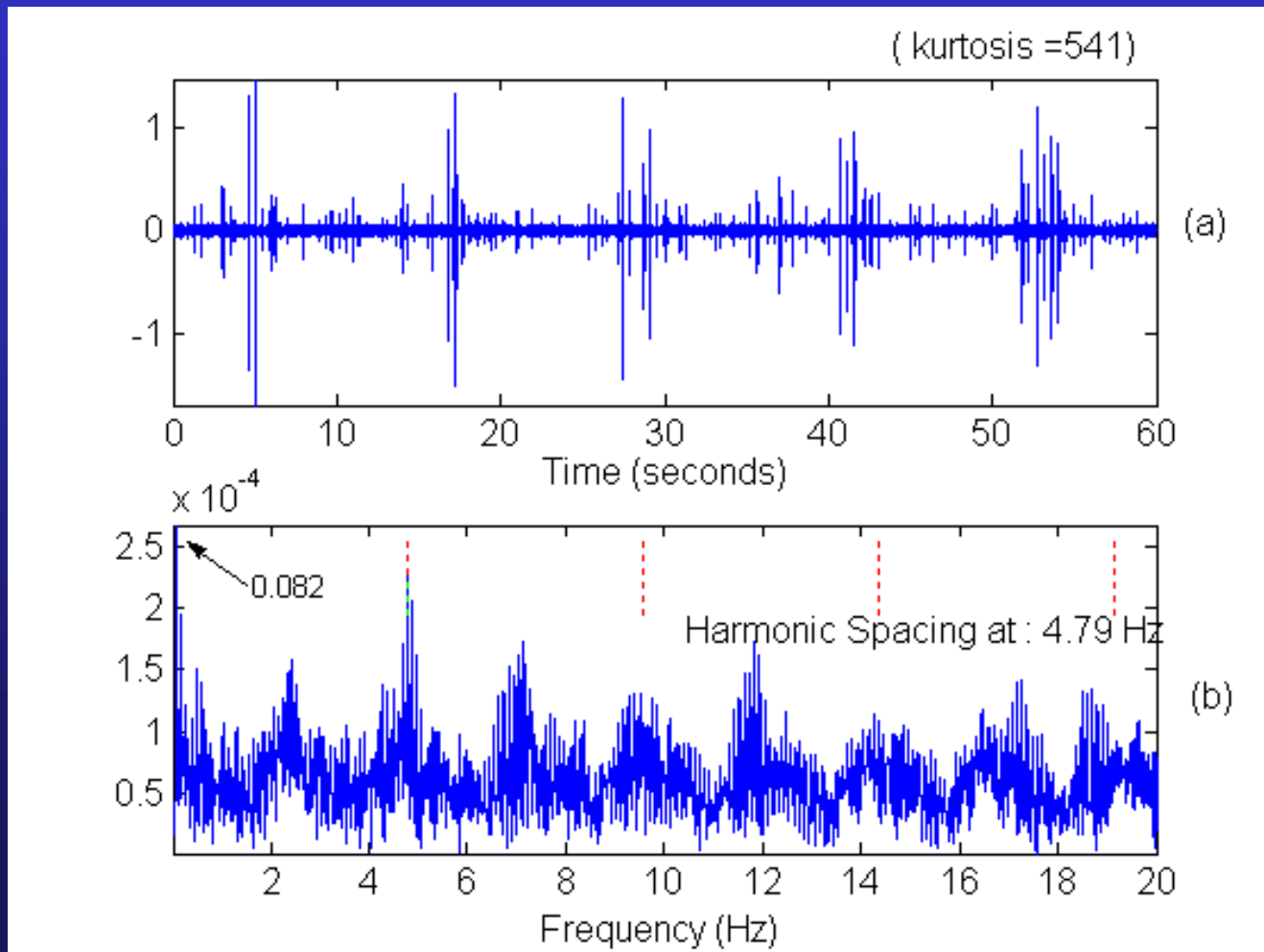
**Total
signal**

**Deterministic
part (gears)
(note scale)**

**Random
part
(bearings)**



Increased kurtosis from SK filtration (gearmesh signals removed)



Note that extremely high kurtosis indicates that it is not an absolute measure of severity. Fault could have been detected at a very early stage

Envelope spectrum showing harmonics of (half) ballpass frequency modulated at rotation speed



GEAR DIAGNOSTICS - METHODS

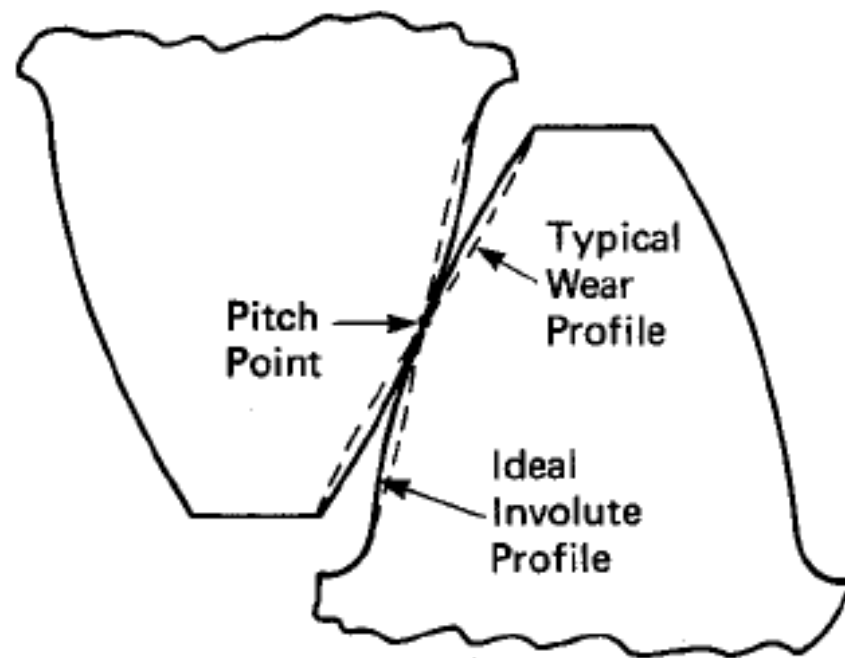
1. **SPECTRUM ANALYSIS** - Useful for detecting changes in the harmonics of toothmesh frequency (uniformly distributed faults)
2. **CEPSTRUM ANALYSIS** - Useful for detecting changes in sideband patterns (non-uniformly distributed faults)
3. **TIME SYNCHRONOUS AVERAGING** - Separates the signal corresponding to one gear from all others and background noise
4. **TOOTHMESH SIGNAL DEMODULATION** - Shown to be sensitive to early tooth root cracks
5. **TRANSMISSION ERROR ANALYSIS** - More direct representation of what is going on at the toothmesh than external vibration signals
6. **WAVELET ANALYSIS** - Better time resolution at high frequencies, but difficult to interpret



UNIFORM ERRORS

- Tooth deflection under load
For constant load is same for each tooth pair. Therefore toothmesh frequency and harmonics are affected. This is load sensitive, so spectrum comparisons must be for same load.
- Mean geometric profile errors
From initial manufacture and wear. By definition this is same for each tooth pair. Therefore toothmesh frequency and harmonics are affected. This is only weakly load sensitive.
- Uniform wear
Gives change in harmonics of toothmesh frequency under constant load conditions. First indication at second harmonic of gearmesh frequency

EFFECT OF WEAR



Because the sliding velocity is zero at the pitch circles and finite on either side, there is a tendency for a “double-scalloped” wear pattern as shown, which initially gives more increase in the second harmonic of the toothmesh frequency



Variations Between Teeth

At rotational harmonics other than toothmesh.
Harmonic spacing indicates which gear has caused change. Can be further subdivided:

- Slow variations, e.g. runout, distortion. Low harmonics and sidebands around toothmesh are affected.
- Local faults, e.g. cracks, spalls. Wide distribution of harmonics results.
- Random errors, e.g. Random tooth spacing error. Wide distribution of harmonics results.
- Systematic errors, e.g. “Ghost components”, from gear cutting machine.



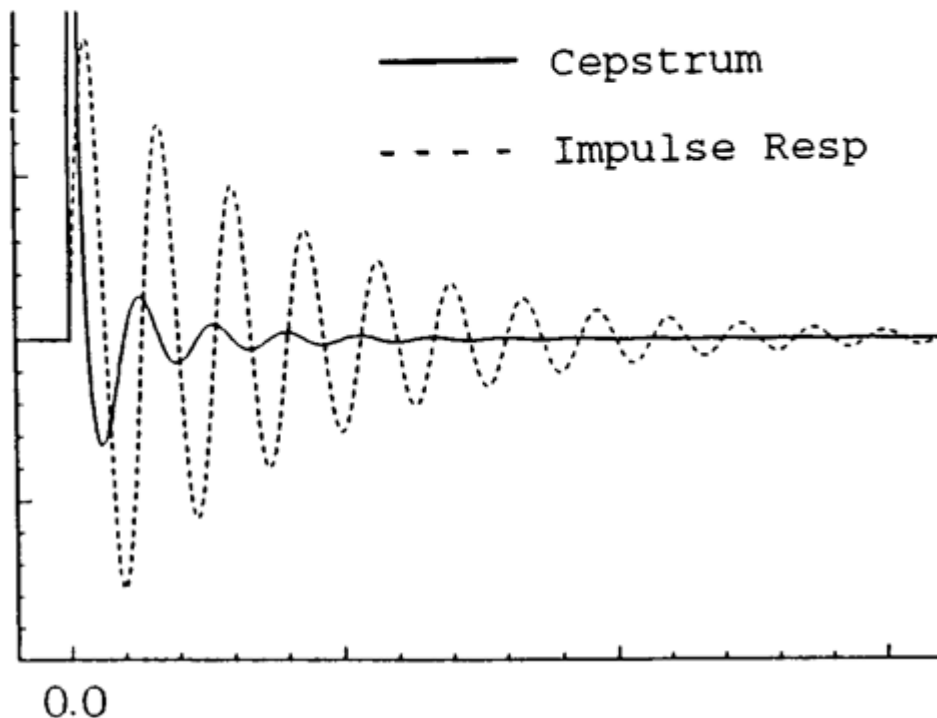
Operational modal analysis using the cepstrum

- Forcing and transfer function effects additive in cepstrum for a single input
- They are also separated for a smooth flat input spectrum (impulsive or random)
- Pole/zero parameters can be extracted from response autospectra, and used to update and scale FRFs
- For multiple inputs, New blind source separation techniques give the possibility of extracting the responses to a particular input
- Cepstral techniques then give the scaled FRFs for the resulting SIMO system



Analytical Expression for the Cepstrum (Oppenheim & Schaffer)

Cepstrum vs Impulse Response for an SDOF System



Cepstrum Equations

$$C(n) = \ln(K) \quad , n = 0$$

$$C(n) = -\sum_i \frac{a_i^n}{n} + \sum_i \frac{c_i^n}{n} \quad , n > 0$$

$$C(n) = \sum_i \frac{b_i^{-n}}{n} - \sum_i \frac{d_i^{-n}}{n} \quad , n < 0$$

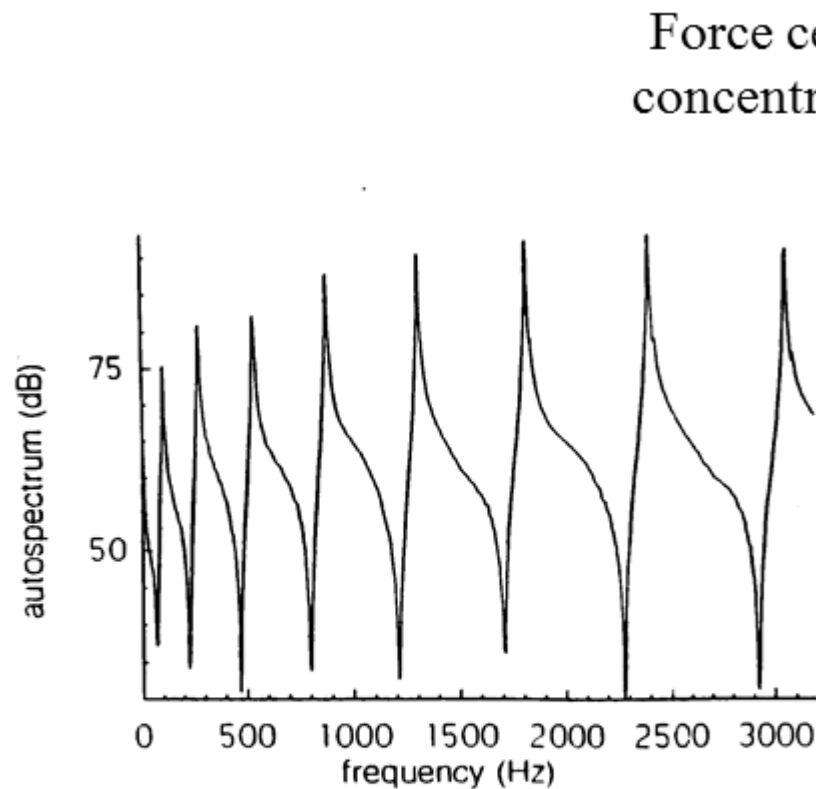
where the a_i and c_i are zeros and poles inside the unit circle

and $1/b_i$ and $1/d_i$ are zeros and poles outside the unit circle.

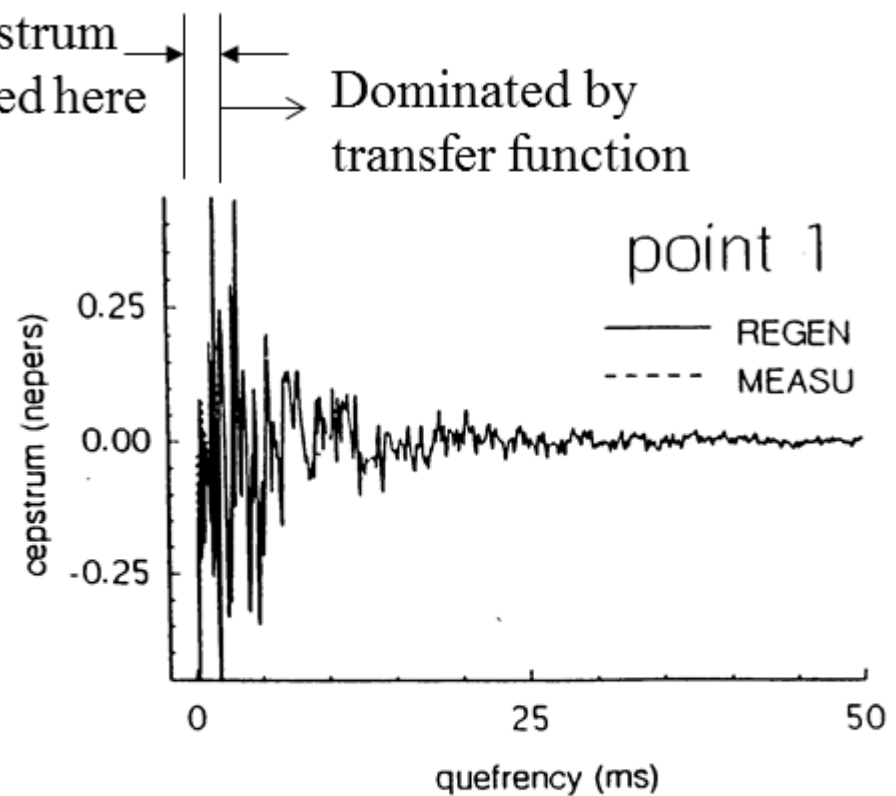
An SDOF system has one conjugate pair of poles c_i which results in an exponentially damped cosine further damped by the hyperbolic function $1/n$



Curve-fitting poles and zeros of transfer function in the response cepstrum



Driving point autospectrum



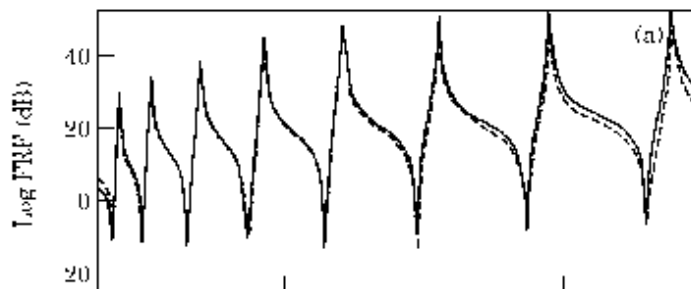
Driving point cepstra

FREE-FREE BEAM, HAMMER EXCITATION

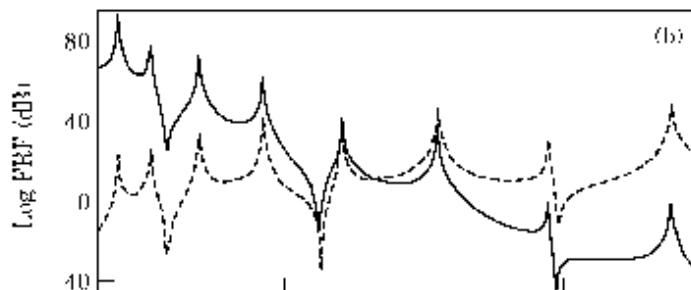


TRUNCATION OF OUT-OF-BAND MODES

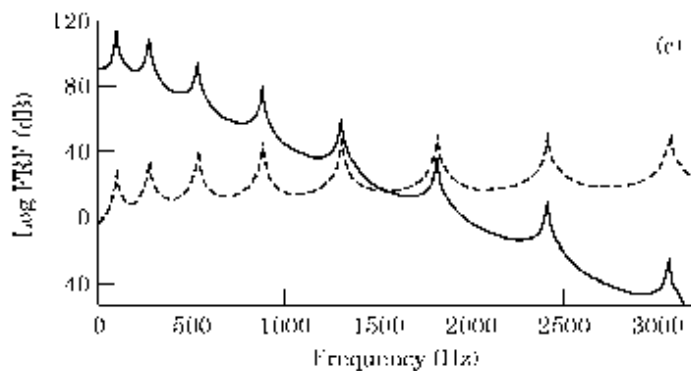
FRFs regenerated from in-band poles and zeros only



Point 1 (driving point).
Poles and zeros balanced



Point 5, typical point.
No. of zeros approx.
half no. of poles



Point 8 (end-to-end).
No zeros



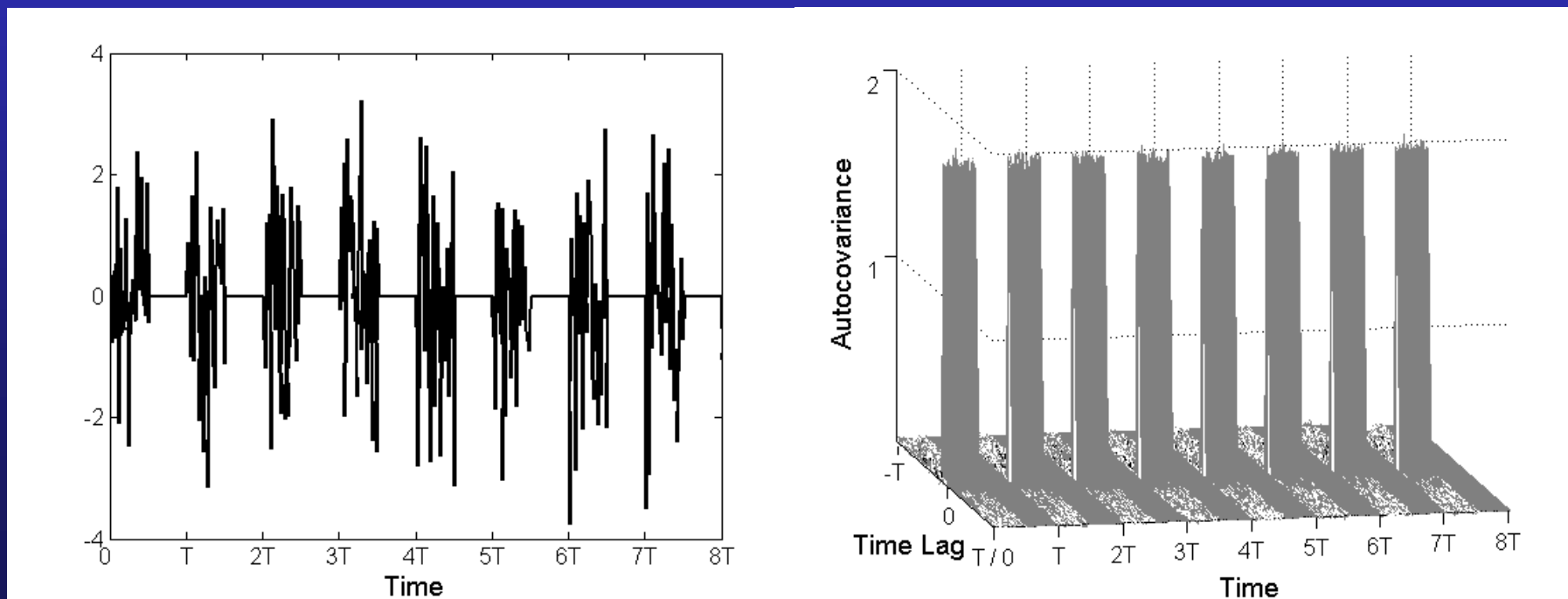
FRF RECONSTRUCTION

- When generating FRFs from in-band poles and zeros only there are two missing factors
- One is an equalisation curve depending on the ratio of poles to zeros
- The other is an overall scaling factor, as this is contained in the zero quefrequency component
- Neither changes greatly with small changes in pole and zero positions, and so can be determined from an earlier measurement, a similar measurement or a finite element model



USE OF CYCLOSTATIONARITY TO OBTAIN SIMO FROM MIMO (David Hanson)

- eg Burst random signal
 - zero mean (1st order)
 - periodic autocovariance (2nd order)
- Spectral correlation is 2D FT of 2D autocovariance

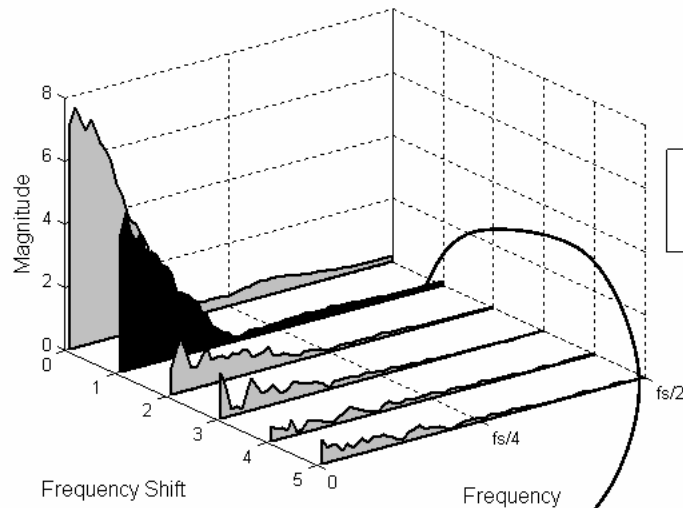


It is also the correlation of the spectrum with itself

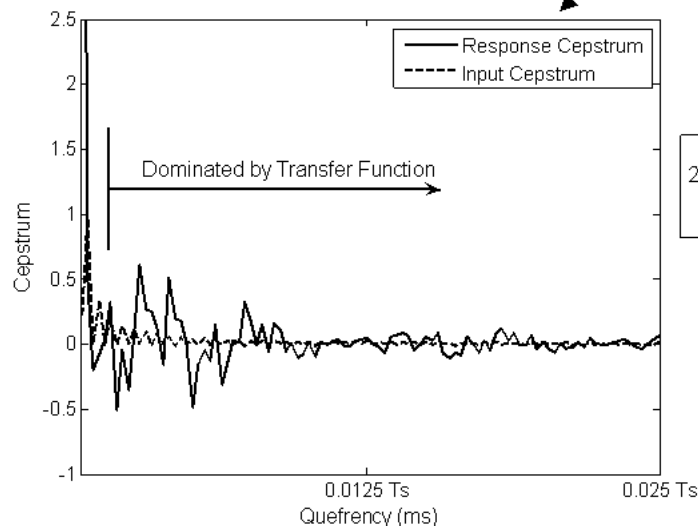
Curve-fitting Cepstrum from Spectral Correlation

Spectral correlation at cyclic frequency α contains structural dynamic information but only that excited by source at cyclic frequency α

The cepstrum obtained from this allows separation of source and transfer path information because of single source



1. Calculate cyclic spectrum to separate contributions to each response from the cyclostationary excitation



2. Use cepstrum to separate transmission path from source



OBTAIN CEPSTRUM FROM CYCLIC SPECTRUM

Starting with the system equation:

$$Y(f) = H(f)X(f)$$

and defining the cyclic spectral density of the response as:

$$S_y^\alpha(f) = \lim_{W \rightarrow \infty} E \left\{ Y_W(f) Y_W^*(f - \alpha) \right\}$$

we get

$$S_Y^\alpha(f) = H(f)H^*(f - \alpha)S_x^\alpha(f)$$

Taking the log and inverse Fourier transform to obtain the cepstrum

$$C_y^\alpha(\tau) = C_h(\tau) + C_h(-\tau)e^{j2\pi\alpha\tau} + C_x^\alpha(\tau)$$

Impulsive force has flat spectrum and short cepstrum, so:

$$C_y^\alpha(\tau) \approx C_h(\tau) + C_h(-\tau)e^{j2\pi\alpha\tau}, |\tau| > \tau_0$$

and if the system is minimum phase

$$C_h(-\tau) = 0$$

so

$$C_y^\alpha(\tau) \approx C_h(\tau), \tau > \tau_0$$



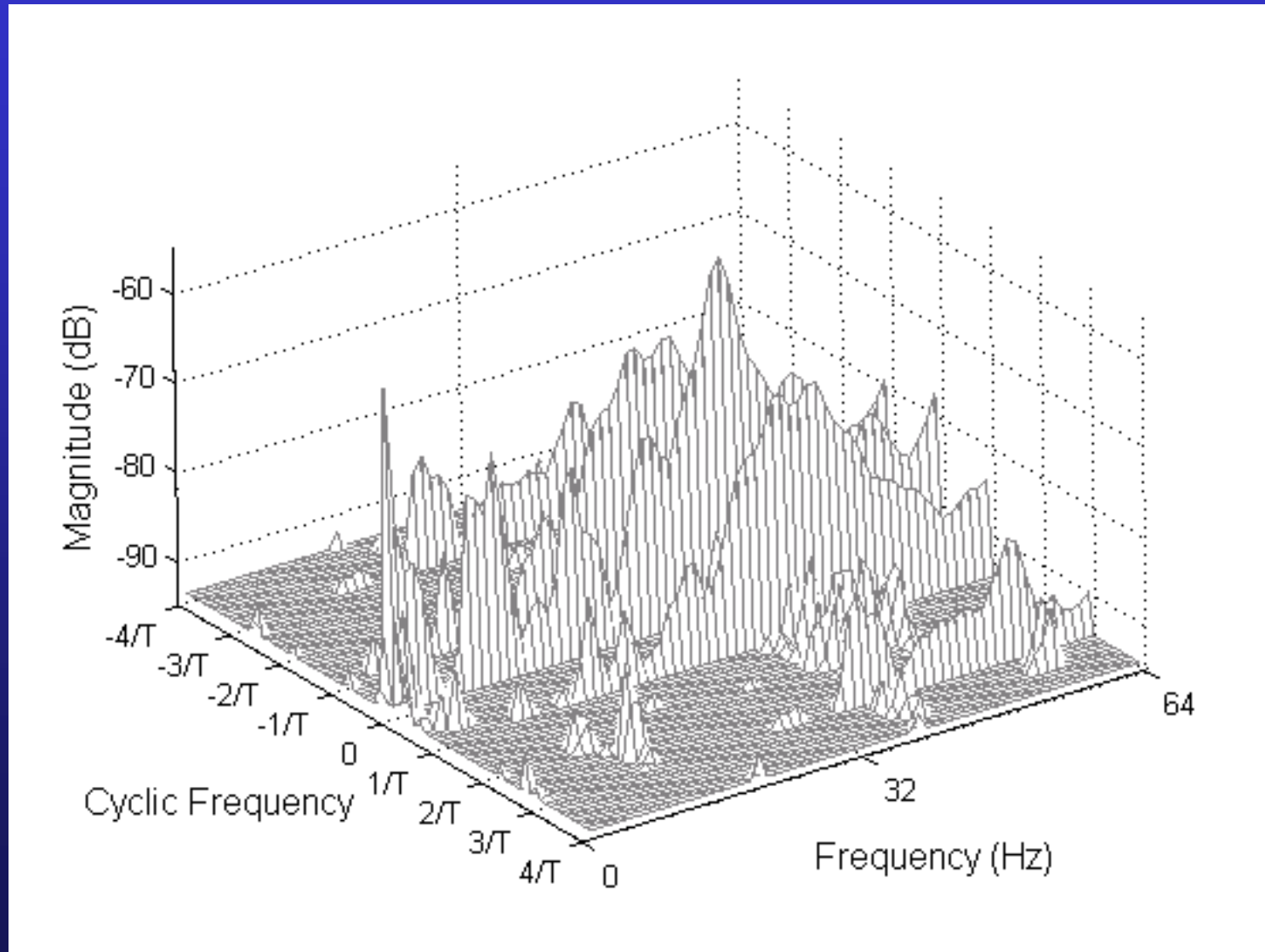
Transperth B Series Railcar

Excited by burst random input from shaker
Supported on elastomeric mounts



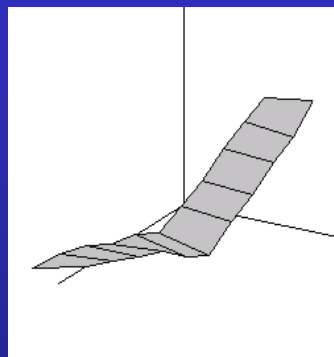


TYPICAL CYCLIC SPECTRA

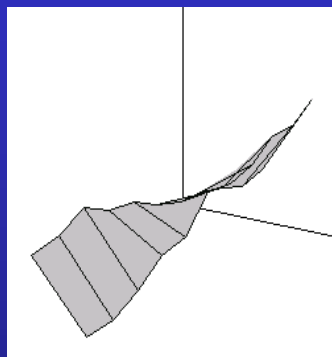




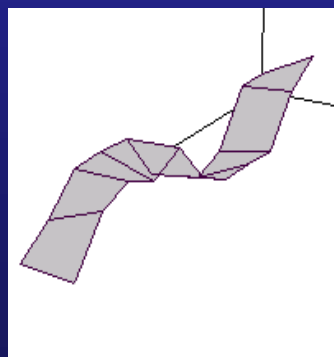
Transperth B Series Railcar OMA Results



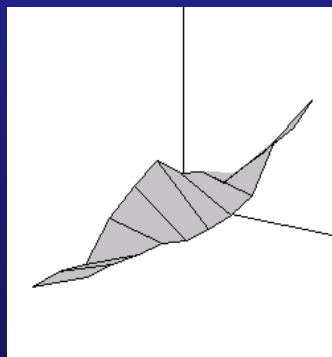
12Hz



16Hz



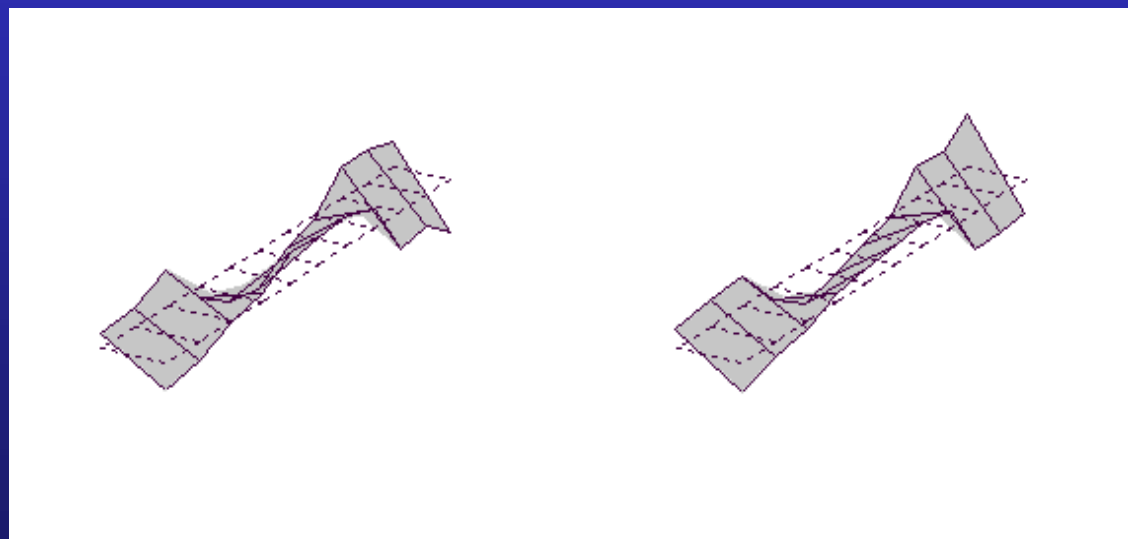
21Hz



26Hz

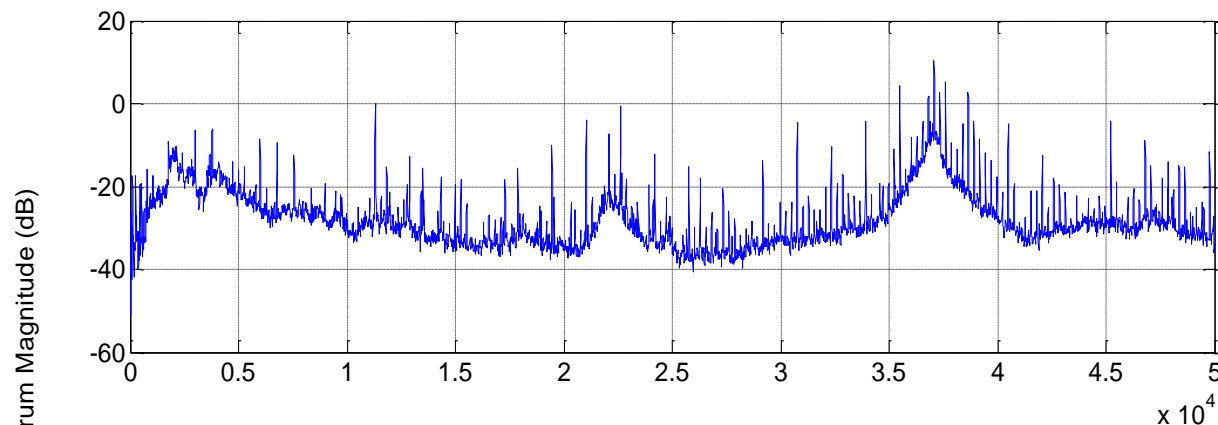
OMA

EMA

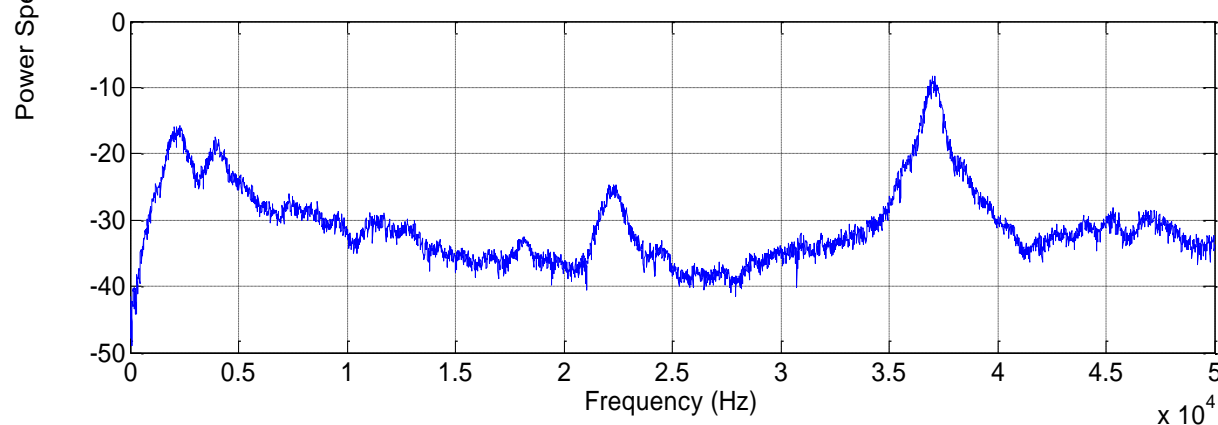




Potential application to machine structural dynamics – gas turbine engine



Total (Raw) signal



Residual signal after editing the Cepstrum

Removal of discrete frequencies – useful for OMA



CONCLUSION

- **Diagnostics involves separating the different signal components, eg discrete frequency from random**
- **Several viable methods available with different pros and cons**
- **Many other techniques available for enhancing various features of faults, for example in bearings and gears**
- **Another useful separation is of forcing function from transfer function for each source and path**
- **Blind determination of transfer functions (system identification) useful to detect faults due to structural change rather than forcing function**
- **Cepstrum useful for many of these functions**