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2012 Annual Conference of the
Prognostics and Health Management Society

An introduction to Prognosis, Uncertainty Representation, and Risk Measures

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1.1) PHM, Fault Diagnosis and Failure Prognosis

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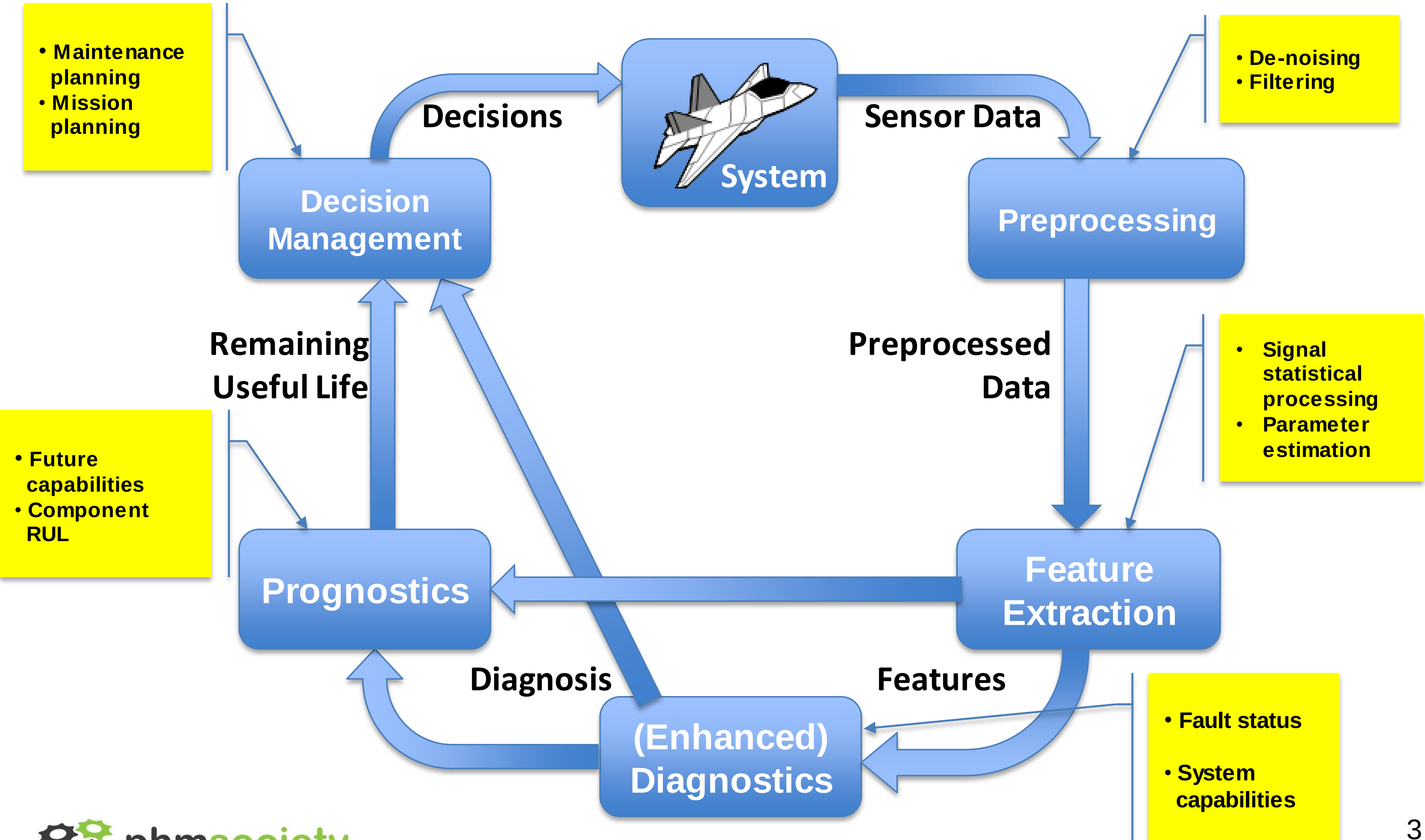

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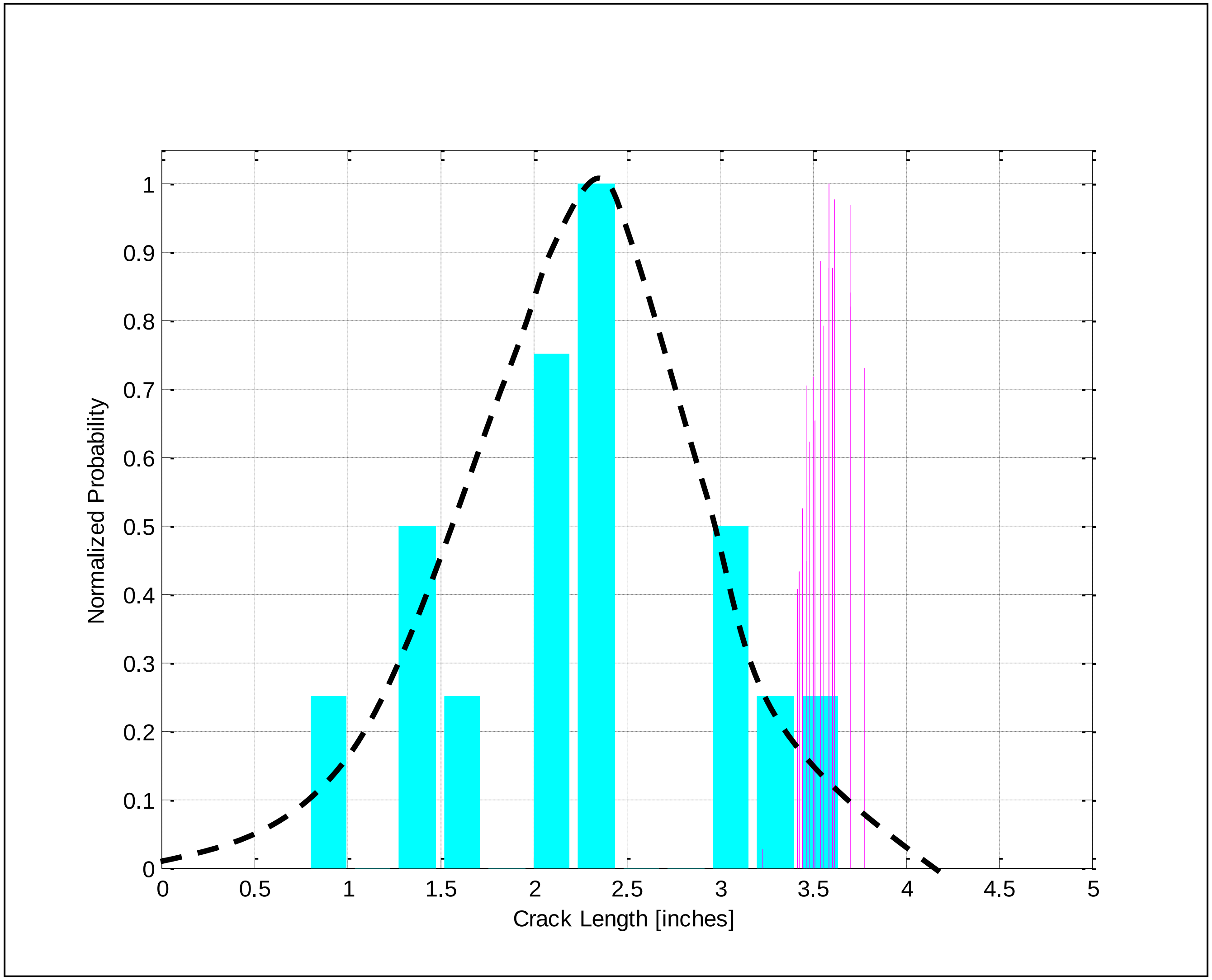
* N. Scott Clements (Ph.D.); PHM 2011, Montreal, Canada

1.1) PHM, Fault Diagnosis and Failure Prognosis



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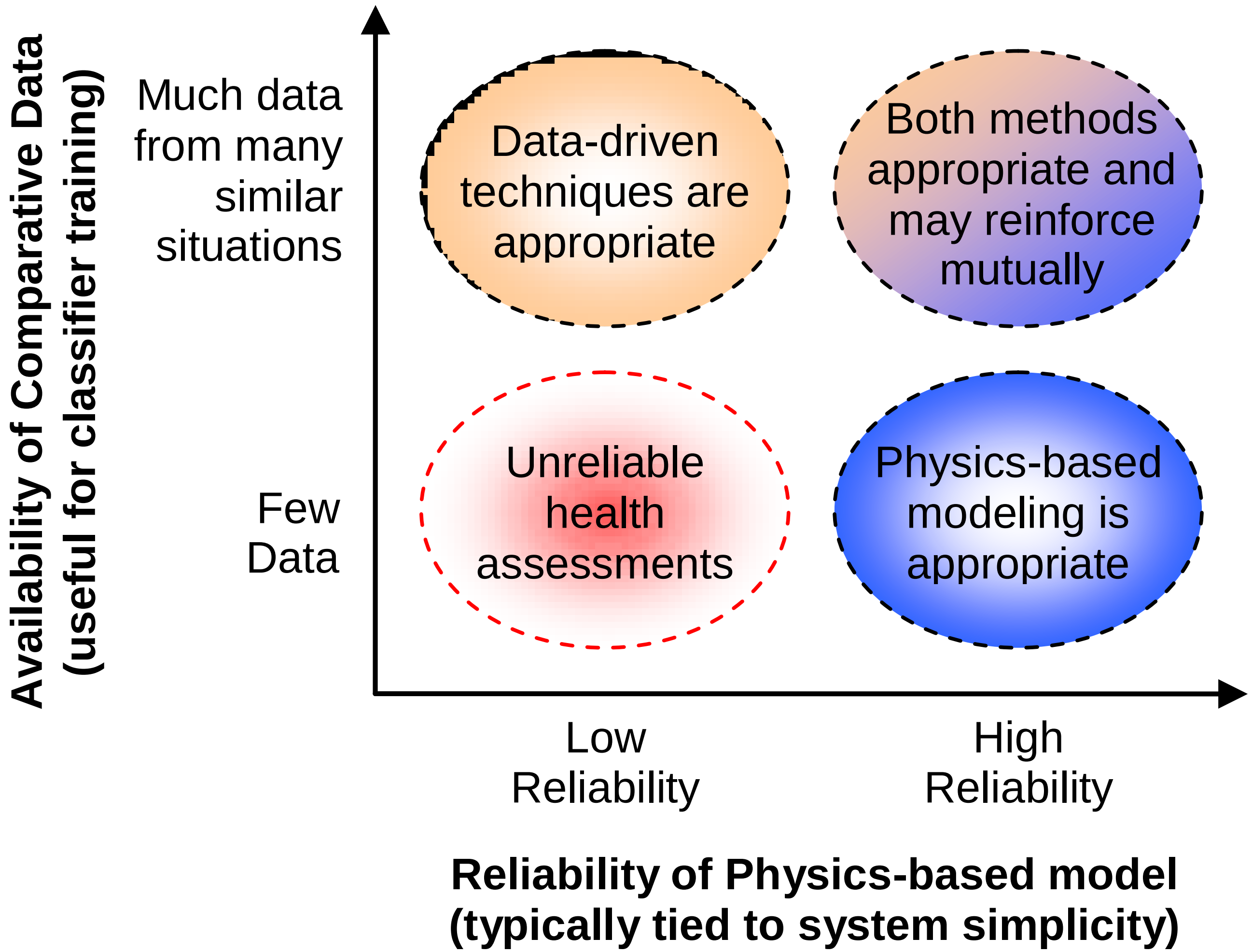
1.2) Process Monitoring: Virtual Sensors and PLS



Source: **Adapted from Inman et al. (2005), p. 6**

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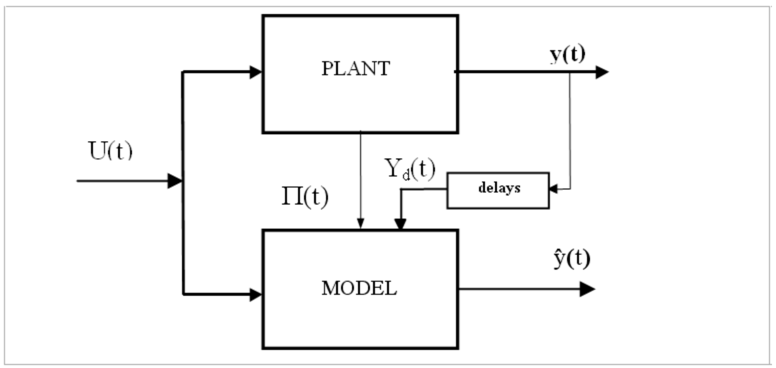
1.2) Process Monitoring: Virtual Sensors and PLS

$$\mathbf{U(t) = [u_1(t)\ u_1(t\text{-}1) \ldots\ u_2(t)\ u_2(t\text{-}1)\ \ldots\ u_r(t)\ u_r(t\text{-}1) \ldots\ ]^T}$$

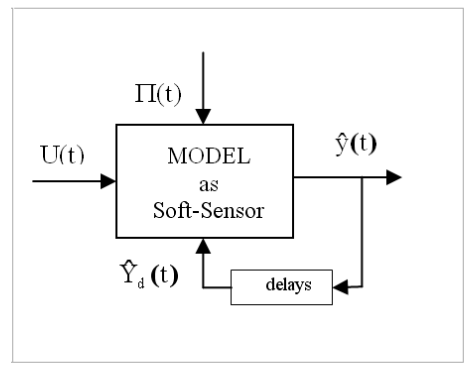
$$\mathbf{\Pi(t) = [\eta_1(t)\ \eta_1(t\text{-}1) \ldots\ \eta_2(t)\ \eta_2(t\text{-}1)\ \ldots\ \eta_p(t)\ \eta_p(t\text{-}1) \ldots\ ]^T}$$

$$\mathbf{Y_d(t) = [y(t\text{-}1)\ y(t\text{-}2)\ \ldots\ y(t\text{-}d)]^T}$$

$$x(t) = \begin{bmatrix} U(t) \\ \Pi(t) \\ Y_d(t) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_q \end{bmatrix}$$



Identification of a dynamic model for $y(t)$ using controls and measured disturbances $u(t)$, other plant outputs $\eta(t)$, and delayed plant outputs $y(t\text{-}d)$



Use of the dynamic model as soft-sensor in the absence of measurement $y(t)$ due to unavailable sensor signal

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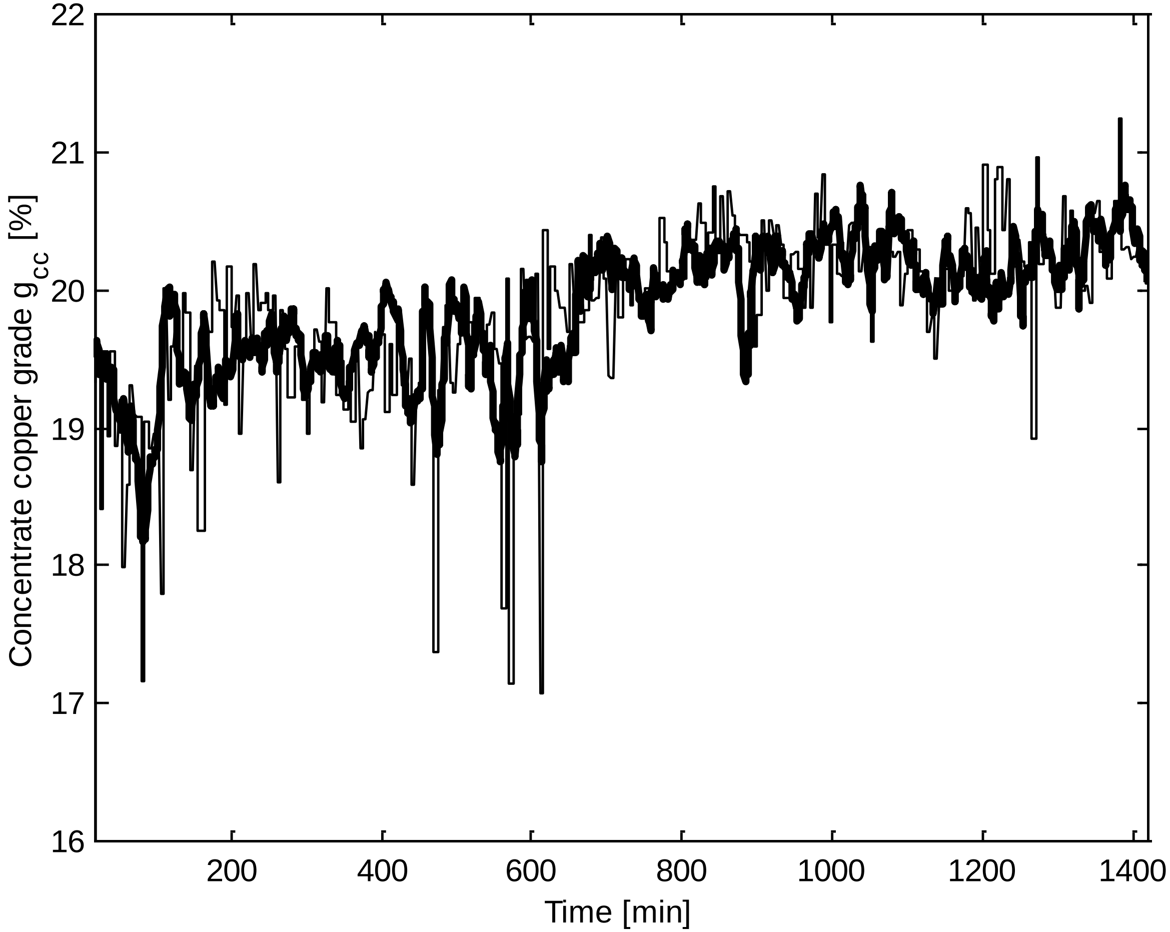
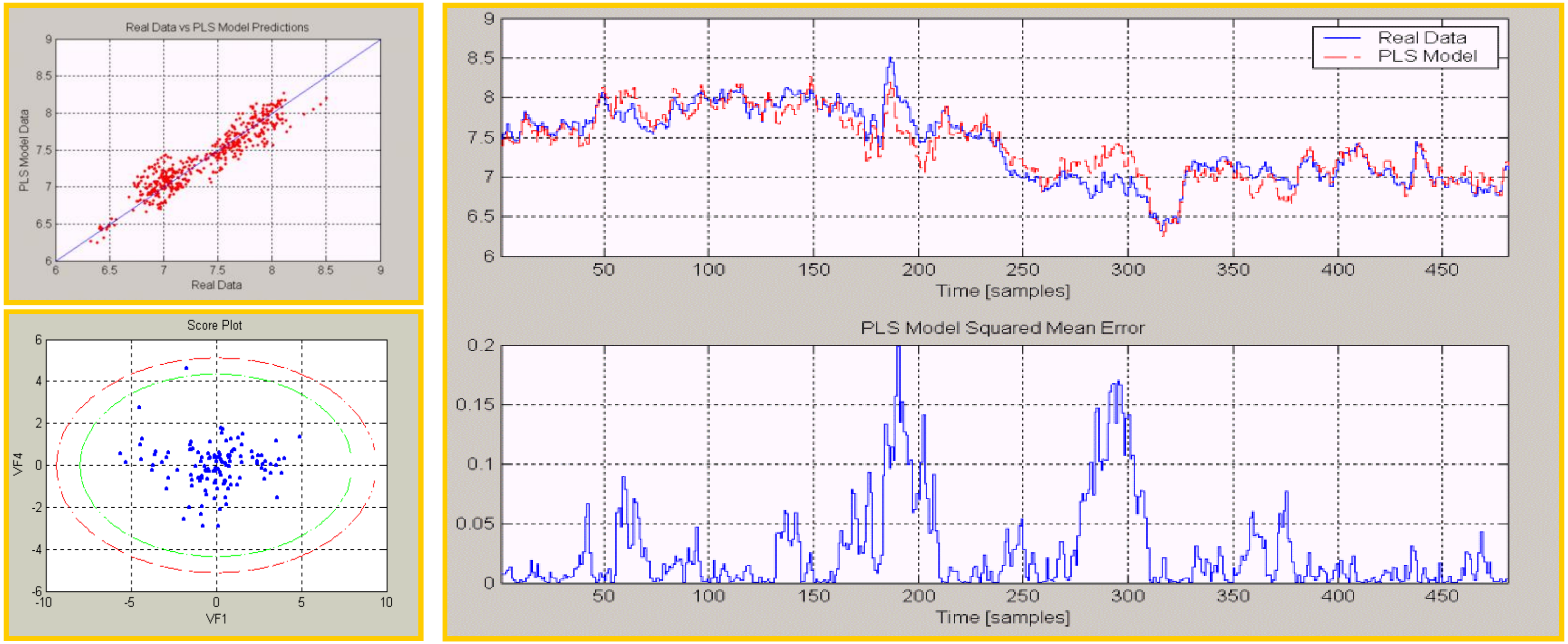
1.2) Process Monitoring: Virtual Sensors and PLS

$$g_{cc}(t) = 0.498 \cdot g_{cc}(t-2) + 0.217 \cdot g_{cf}(t) - 0.046 \cdot L_p(t) - 0.217 \cdot \tau(t-2) - 0.115 \cdot g_{ff}(t) - 0.108 \cdot g_{ff}(t-7)$$



1.2) Process Monitoring: PLS

- Some examples from a rougher flotation plant, where the copper grade is the controlled variable (g_{cc}[%]):



*** CONTAC Ingenieros Ltda., Software "SCAN"**

1.2) Process Monitoring: PLS

- Recursive algorithm that can find directions of "maximum explicability“, building a relation between a group of input variables and a set of output variables.

- Method that eases Model Structure Determination and Parameter Estimation in linear-in-the-parameters models.

$$X = \sum_{i=1}^{A} t_i p_i^T + E_x(A) \quad \text{and} \quad Y = \sum_{i=1}^{A} t_i c_i^T + E_y(A)$$

$$Y = XB \quad , B = [w_1 \cdots w_A] \cdot \left([p_1 \cdots p_A]^T \cdot [w_1 \cdots w_A] \right)^{-1} \cdot [c_1 \cdots c_A]^T$$

- In addition, it allows to statistically characterize the prediction error in multivariate models.
- Off-line estimation technique. Model parameters are assumed to be **constant**!

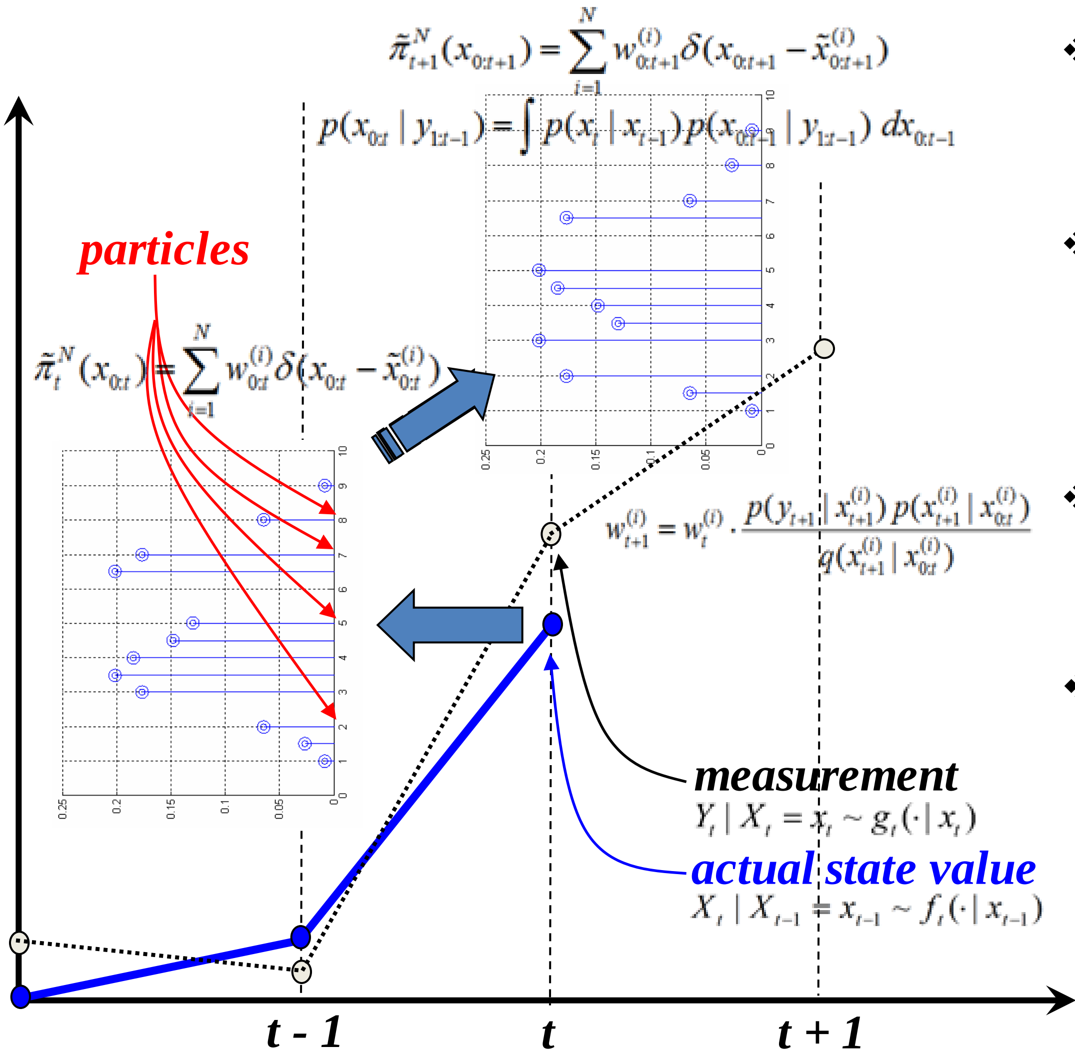
1.3) Parameter Uncertainty and Particle Filters

- **Concept of "Artificial Evolution"**

$$\begin{cases} x(t+1) = f_t(x(t), x_\alpha(t), \omega_1(t)) \\ x_\alpha(t+1) = x_\alpha(t) + \omega_\alpha(t) \\ \text{Features}(t) = h_t(x(t), x_\alpha(t), v(t)) \end{cases}$$

  - f_t and h_t are non-linear mappings.
  - $x(t)$ is the state vector.
  - $\omega_1(t)$ and $v(t)$ are non-Gaussian distributions
  - $x_\alpha(t)$ is an state associated with an unknown model parameter α
  - $\omega_\alpha(t)$ is zero-mean random noise

1.3) Parameter Uncertainty and Particle Filters



❖ Particle: Duple $\{w_t^{(i)}, x_{0:t}^{(i)}\}$, being $x_{0:t}^{(i)}$ a realization of process state *pdf*.

❖ Every particle is associated with an scalar $w_t^{(i)}$, namely the weight

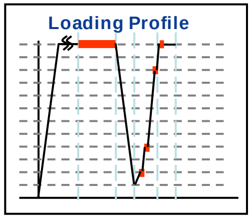
- Sampled version of the PDF

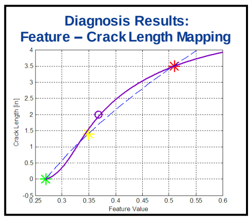
❖ We only need to study the propagation of particles in time!

❖ **<u>Steps</u>:**

- Predict the "*a priori*" PDF, using the model
- Update parameters, given the new measurement

2) Model Uncertainty and PF-based Fault Diagnosis





Structure for Crack Progression Model (FASTRAN / Paris' Equation / ANSYS / Mc Fadden)

$$\frac{dL}{dN} = f(L,N,\theta)$$

Diagnosis Block

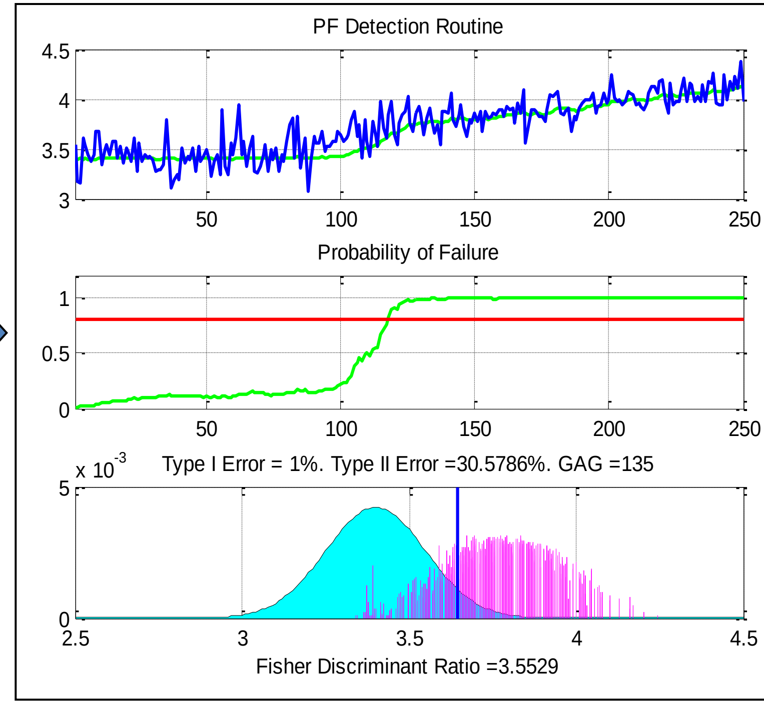
$$\begin{bmatrix} x_{d,1}(t+1)\\ x_{d,2}(t+1)\end{bmatrix} = f_b\left(\begin{bmatrix} x_{d,1}(t)\\ x_{d,2}(t)\end{bmatrix} + n(t)\right)$$

$$x_c(t+1)=x_c(t)+\beta\cdot x_c(t)\cdot x_{d,2}(t)+\omega(t)$$

$$y(t)=x_c(t)+v(t)$$

$$f_b(x)=\begin{cases}[1\quad 0]^T\ , \text{ if } \left\|x-[1\quad 0]^T\right\|\le\left\|x-[0\quad 1]^T\right\| \\ [0\quad 1]^T\ , \text{ else}\end{cases}$$

$$\begin{bmatrix} x_{d,1}(0)\\ x_{d,2}(0)\\ x_c(0)\end{bmatrix} = \begin{bmatrix} 1\\ 0\\ 3.4\end{bmatrix}$$



Baseline Data

Specifications

Type I error = 5% (designed)

Type II Error = 5% (desired)

2) Model Uncertainty and PF-based Fault Diagnosis

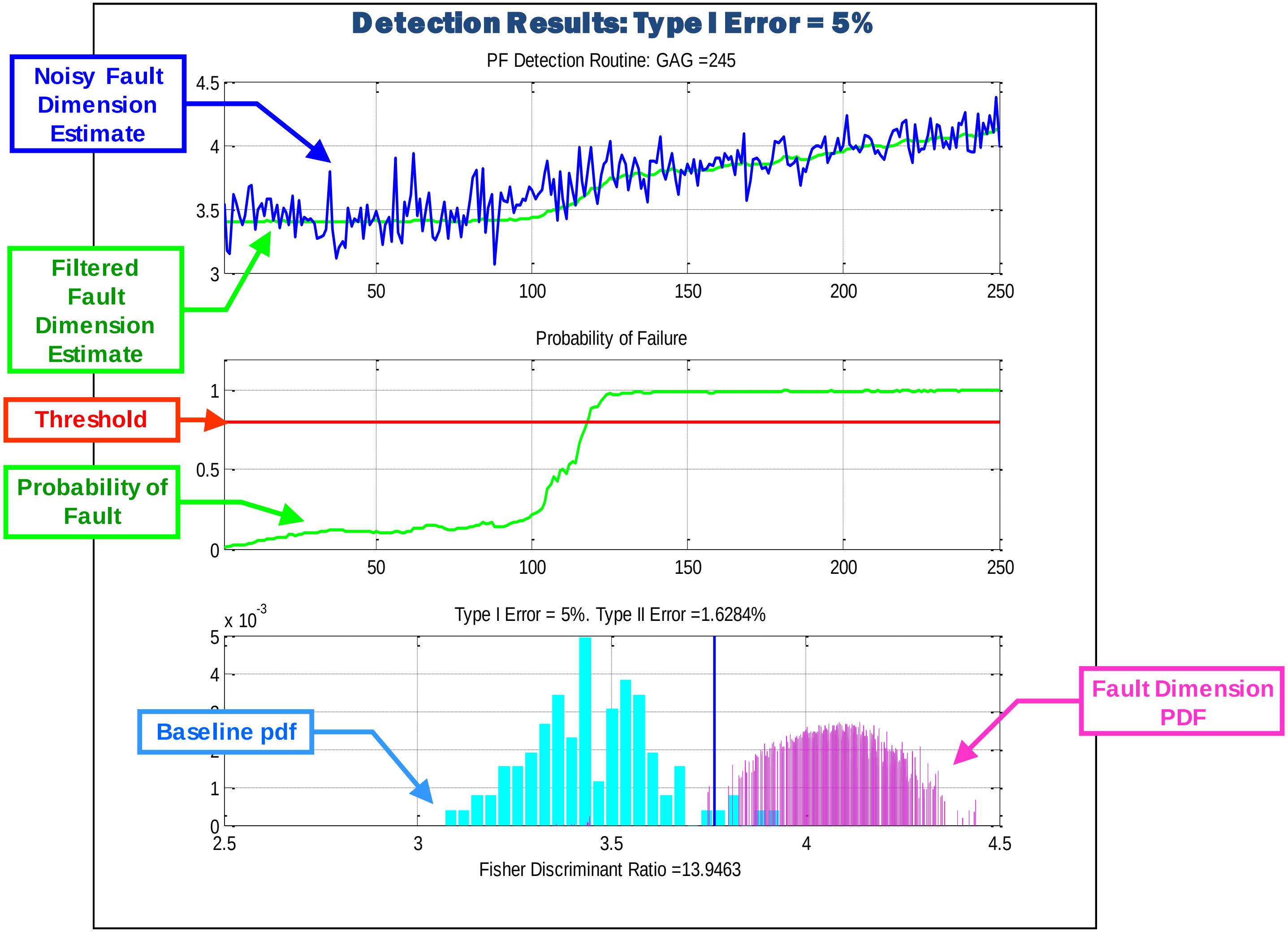
Summary:

- Type I Error (*False Positives*) fixed at 5%
  - Design parameter
- Type II Error (*False Negatives*)

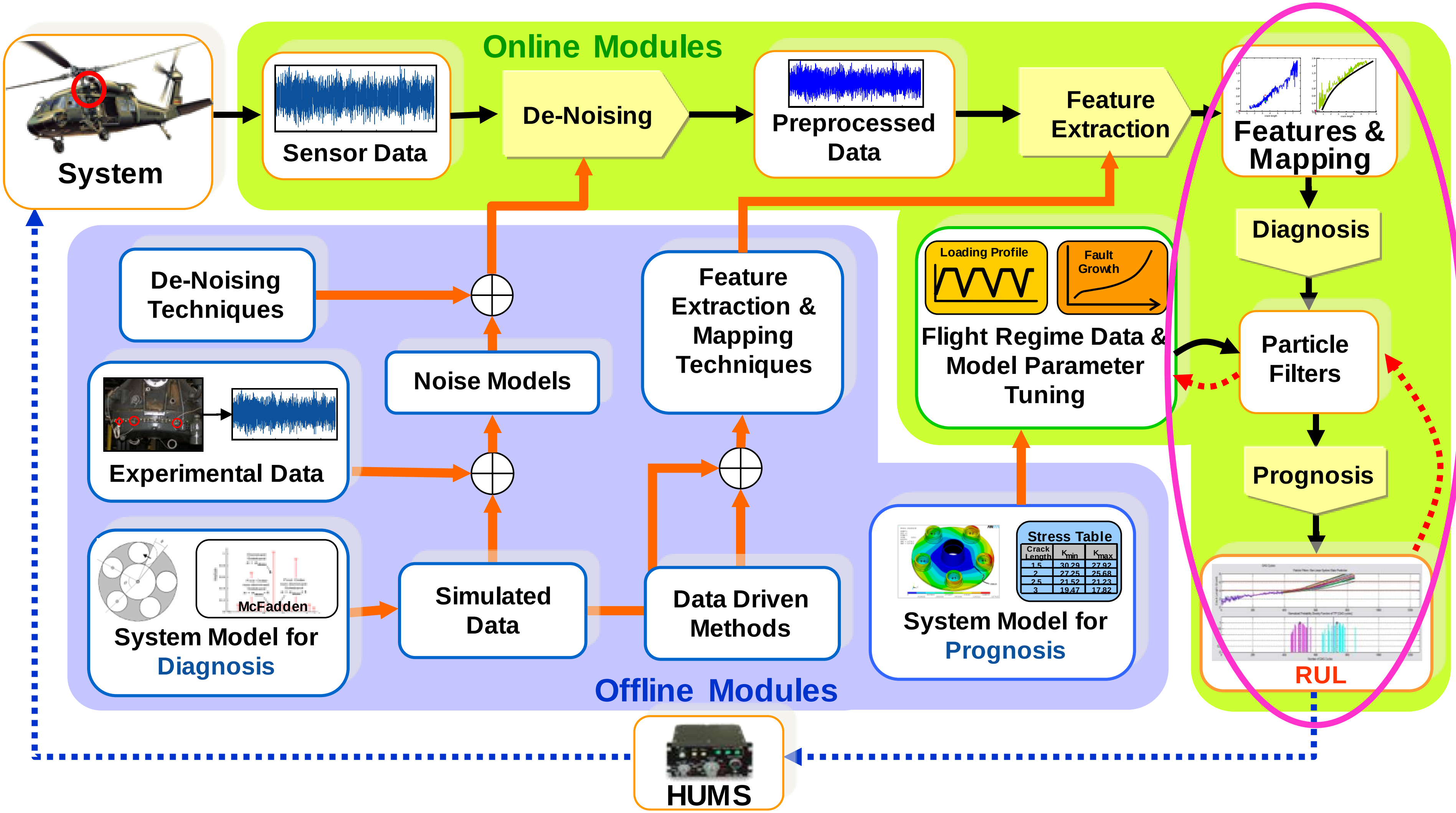
$$1-\sum_i w_T^{(i)} \text{ such that } x_c^{(i)}(T) \geq z_{1-\alpha,\mu,\sigma^2}$$

- Estimated Probability of Fault Condition = $E\left\{x_{d,2}\right\}$
- Fisher's Discriminant Ratio

2) Model Uncertainty and PF-based Fault Diagnosis

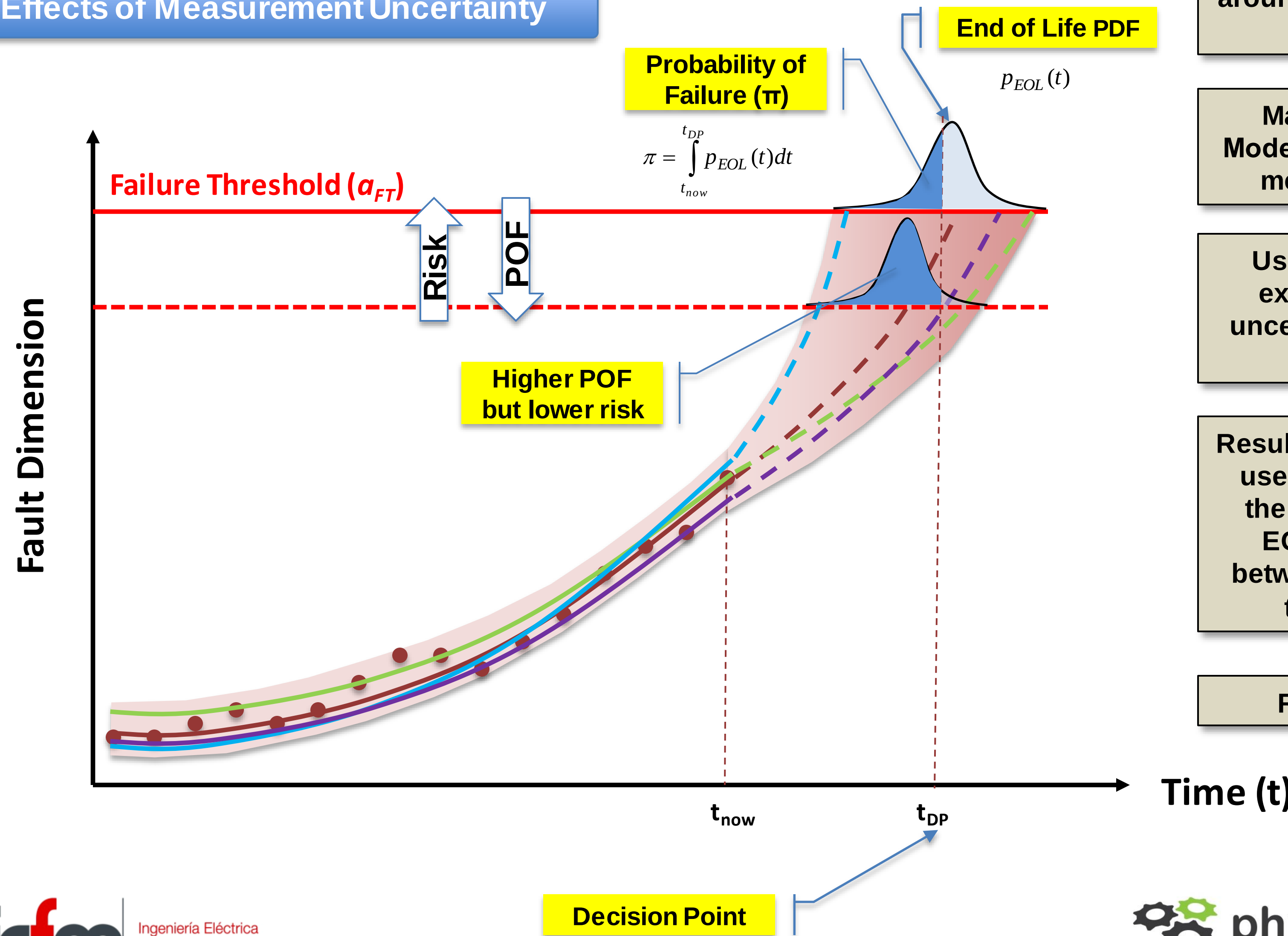


3) PF-based Failure Prognosis



3) PF-based Failure Prognosis

Effects of Measurement Uncertainty



Band of uncertainty around measurement points

Many possible Models may "fit" the measurements

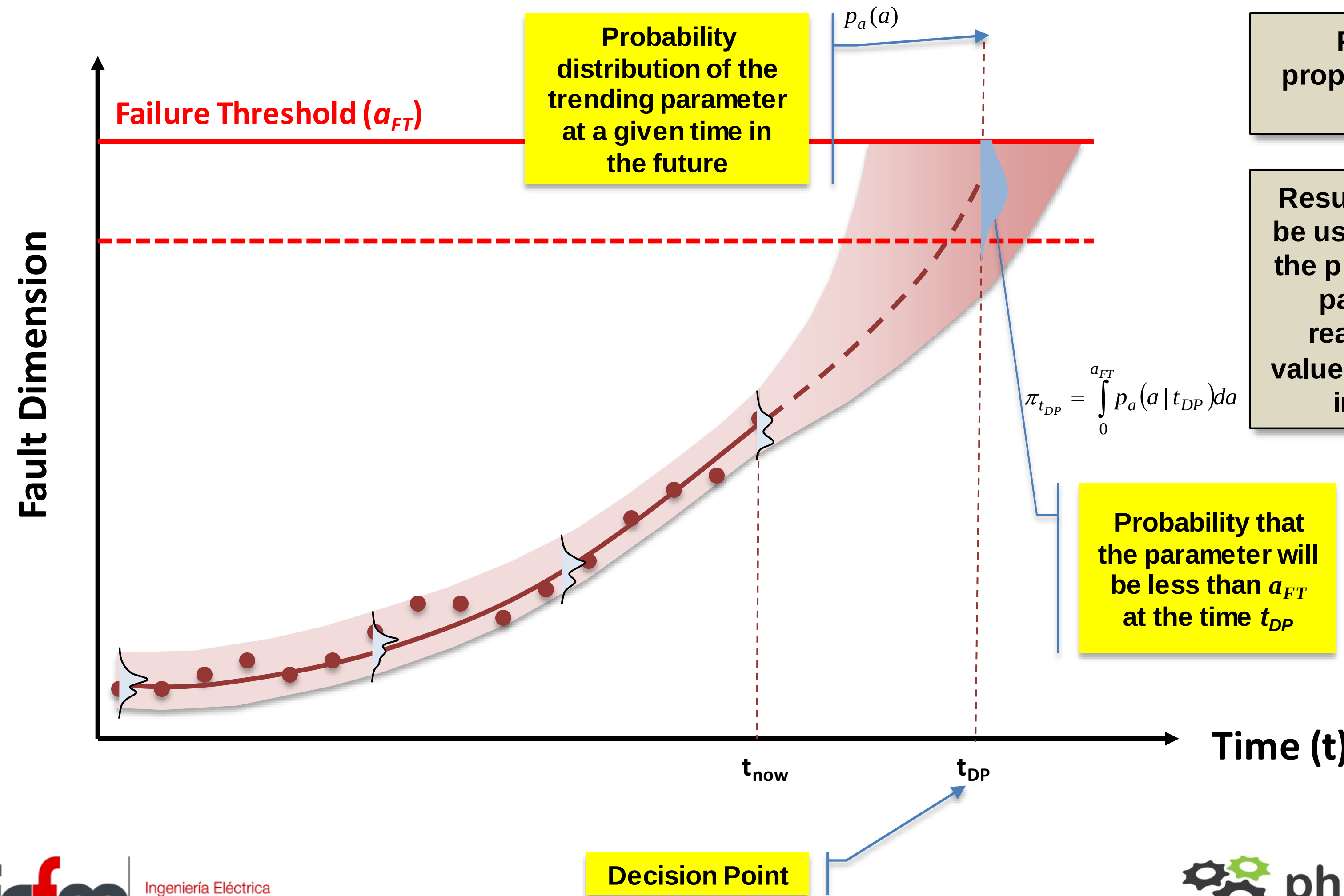
Use statistics to extrapolate the uncertainty into the future

Resulting PDF can be used to determine the probability of EOL occurring between two future time points

Risk vs POF

3) PF-based Failure Prognosis

Effects of Measurement Uncertainty



Can be represented by a PDF describing the initial conditions

PDF is then propagated forward in time

Resulting PDF's can be used to determine the probability that a parameter has reached a given value at a given point in the future

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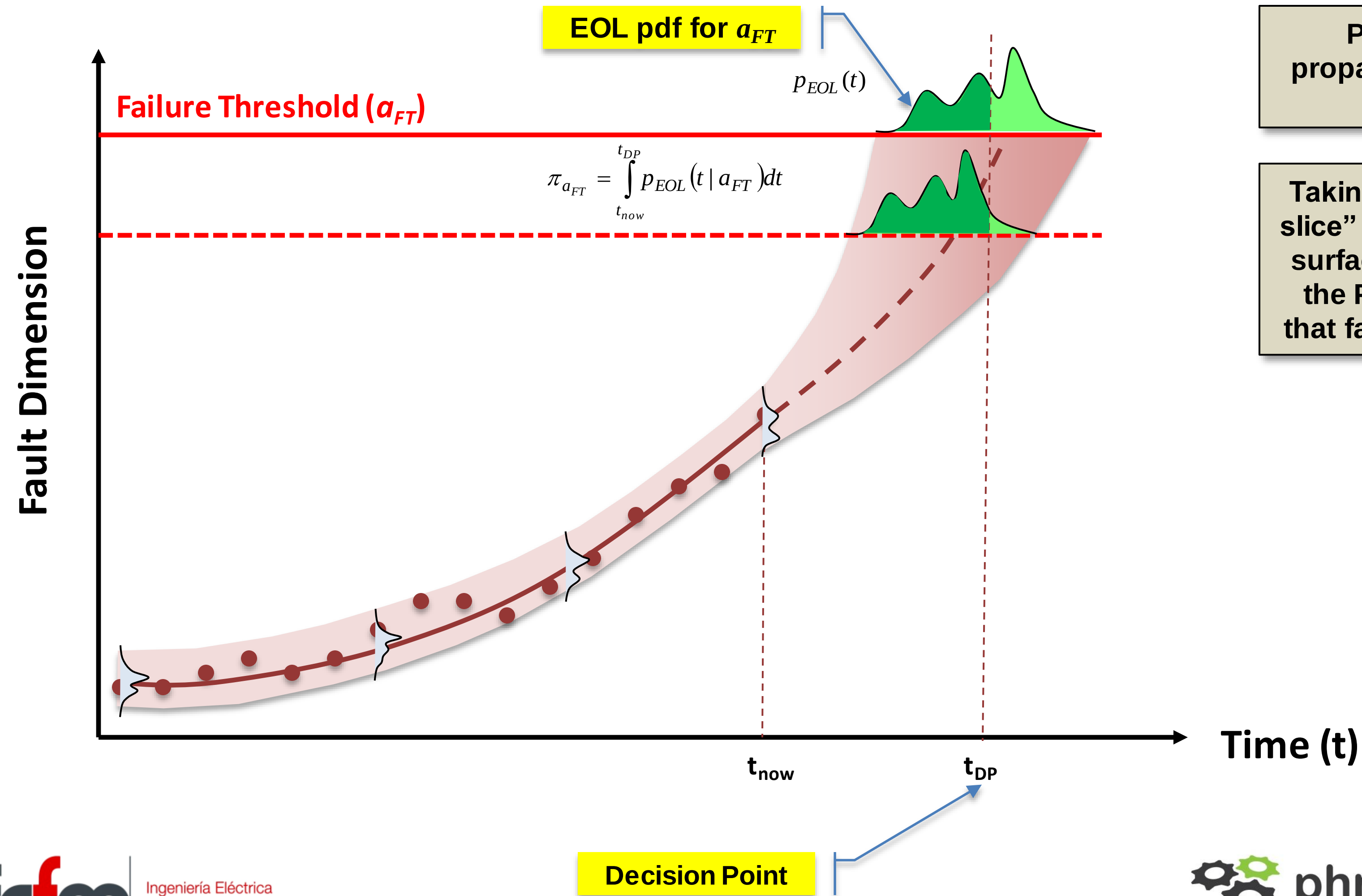
3) PF-based Failure Prognosis

Effects of Measurement Uncertainty



Can be represented by a PDF describing the initial conditions

PDF is then propagated forward in time

Taking a "horizontal slice" of the resulting surface at a_{FT} yields the PDF of EOL at that failure threshold

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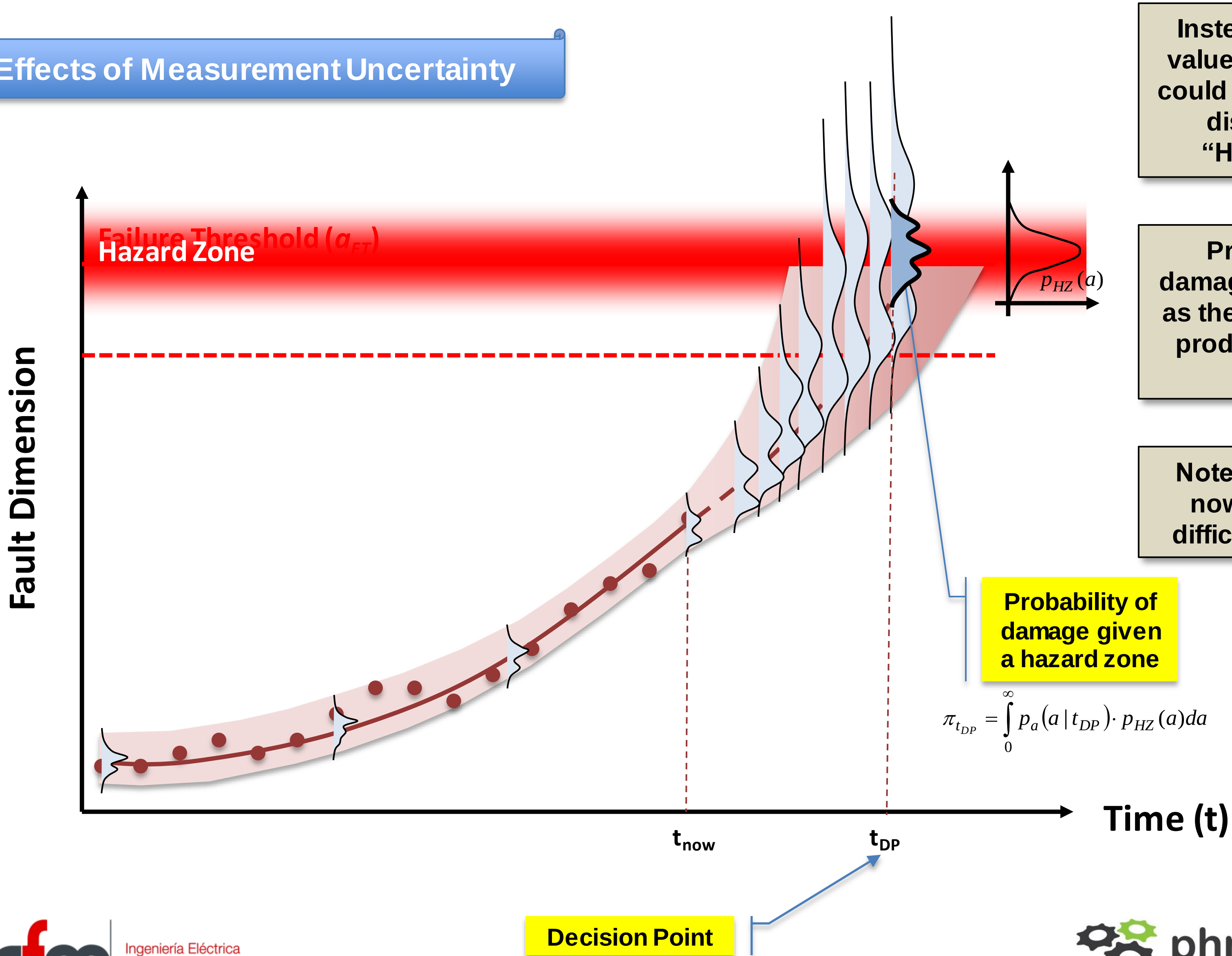
3) PF-based Failure Prognosis

Effects of Measurement Uncertainty

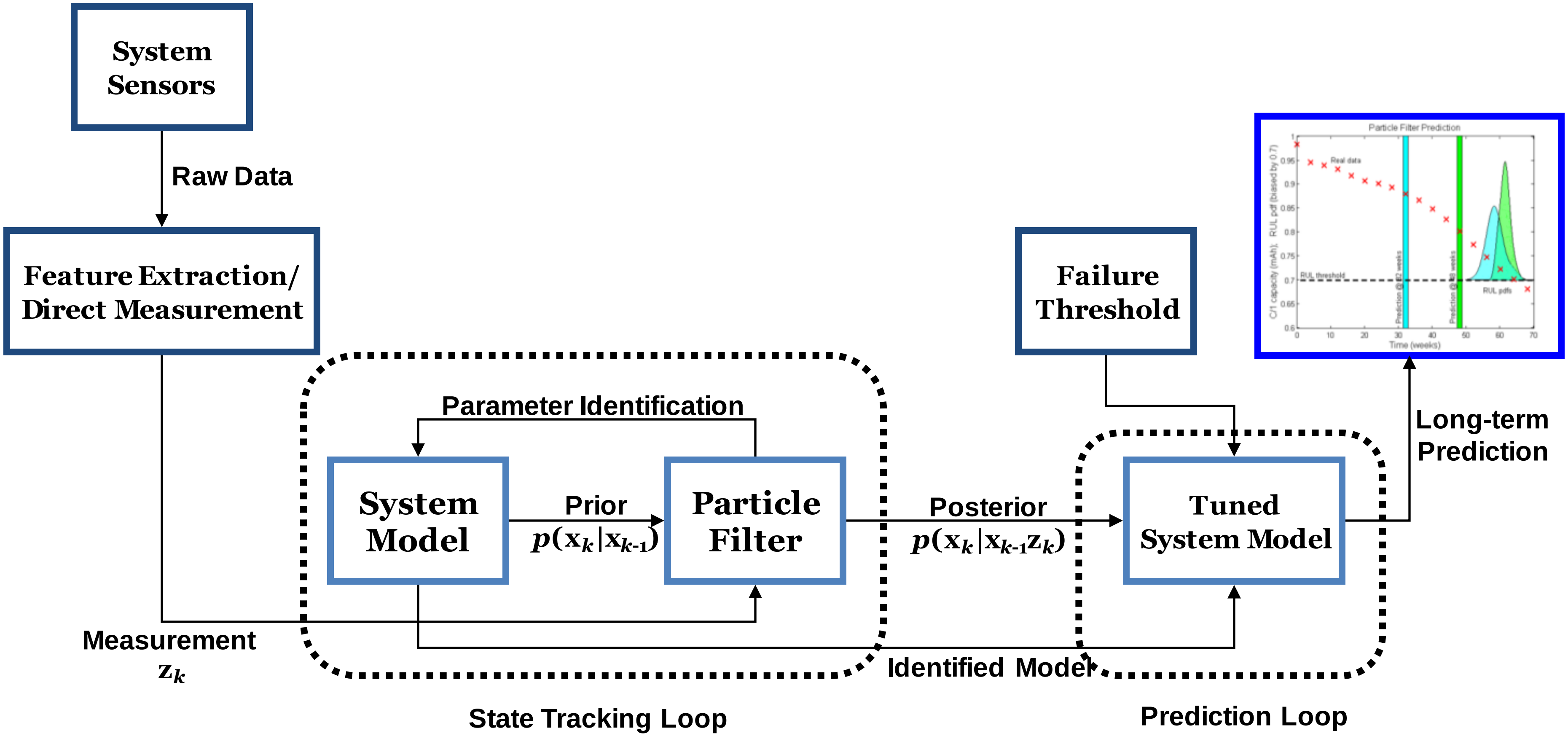


Instead of a single value, the threshold could be defined as a distribution – "Hazard Zone'

Probability of damage is now taken as the integral of the product of the two PDF's

Note that "Risk" is now much more difficult to quantify

3) PF-based Failure Prognosis



3) PF-based Failure Prognosis

- **Dynamic Model for Feature Growth in Time:**

$$\begin{cases} x_1(t+1) = x_1(t) + x_2(t) \cdot F(x_1(t), t, U) + \omega_1(t) \\ x_2(t+1) = x_2(t) + \omega_2(t) \end{cases}$$

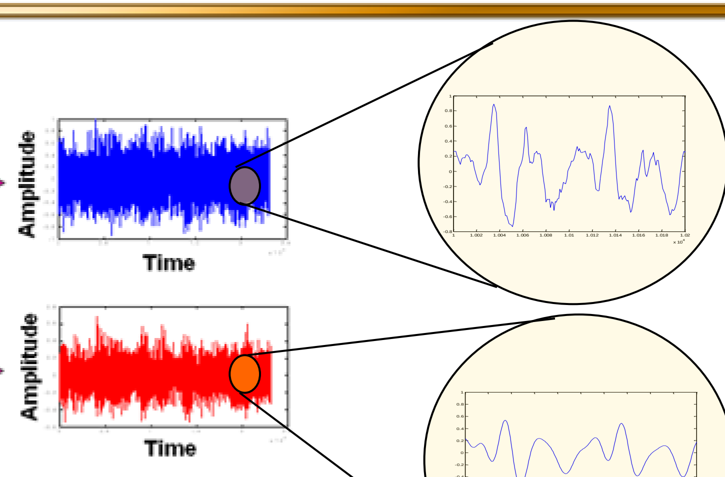
  - $x_1(t)$ is a state representing the fault dimension under analysis
  - $x_2(t)$ is a state associated with an unknown model parameter
  - U are external inputs to the system (load profile, etc.)
  - $F(x(t), t, U)$ is a general time-varying nonlinear function
  - $\omega_1(t)$ and $\omega_2(t)$ are white noises (non necessarily Gaussian)

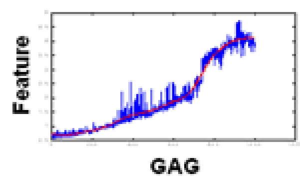
- **Predicted State Density:**

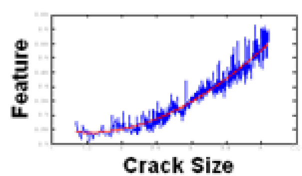
$$\hat{p}(x_{t+k} \mid \hat{x}_{1:t+k-1}) \approx \sum_{i=1}^{N} w_{t+k-1}^{(i)} K\left(x_{t+k} - E\left[x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)}\right]\right)$$

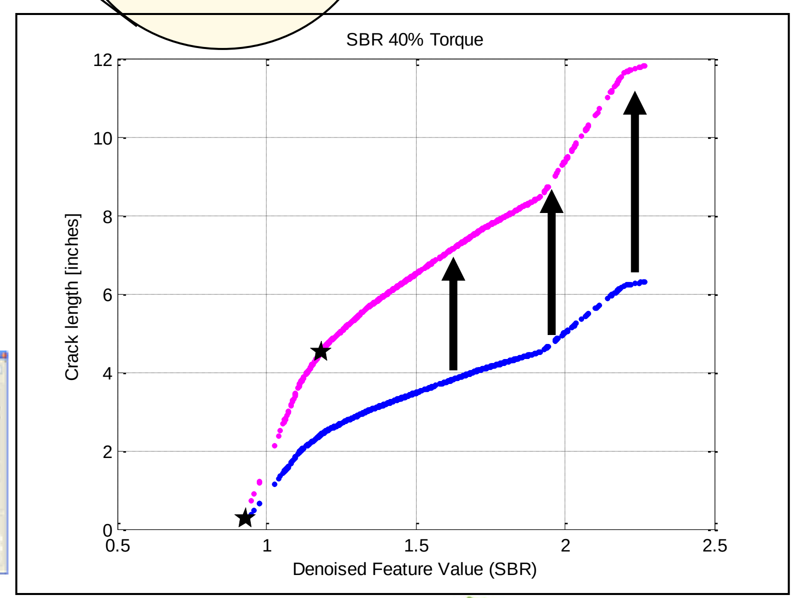
3) PF-based Failure Prognosis

- Raw vibration data
- Data pre-processing TSA, blind deconvolution
- Feature extraction SBR and relSX
- Nonlinear mapping
- Fault diagnosis
- Failure prognosis









3) PF-based Failure Prognosis

PARTICLE FILTERING-BASED FRAMEWORK

- Estimating the Remaining Useful Life (RUL)

➢ **Generation of Long-Term Predictions**

- p-step predictions for a fault indicator
- Prediction entails large-grain uncertainty

$$\tilde{p}(x_{t+p} \mid y_{1:t}) = \int \tilde{p}(x_t \mid y_{1:t}) \prod_{j=t+1}^{t+p} p(x_j \mid x_{j-1}) dx_{t:t+p-1}$$

$$\approx \sum_{i=1}^{N} w_t^{(i)} \int \cdots \int p(x_{t+1} \mid x_t^{(i)}) \prod_{j=t+2}^{t+p} p(x_j \mid x_{j-1}) dx_{t+1:t+p-1}$$

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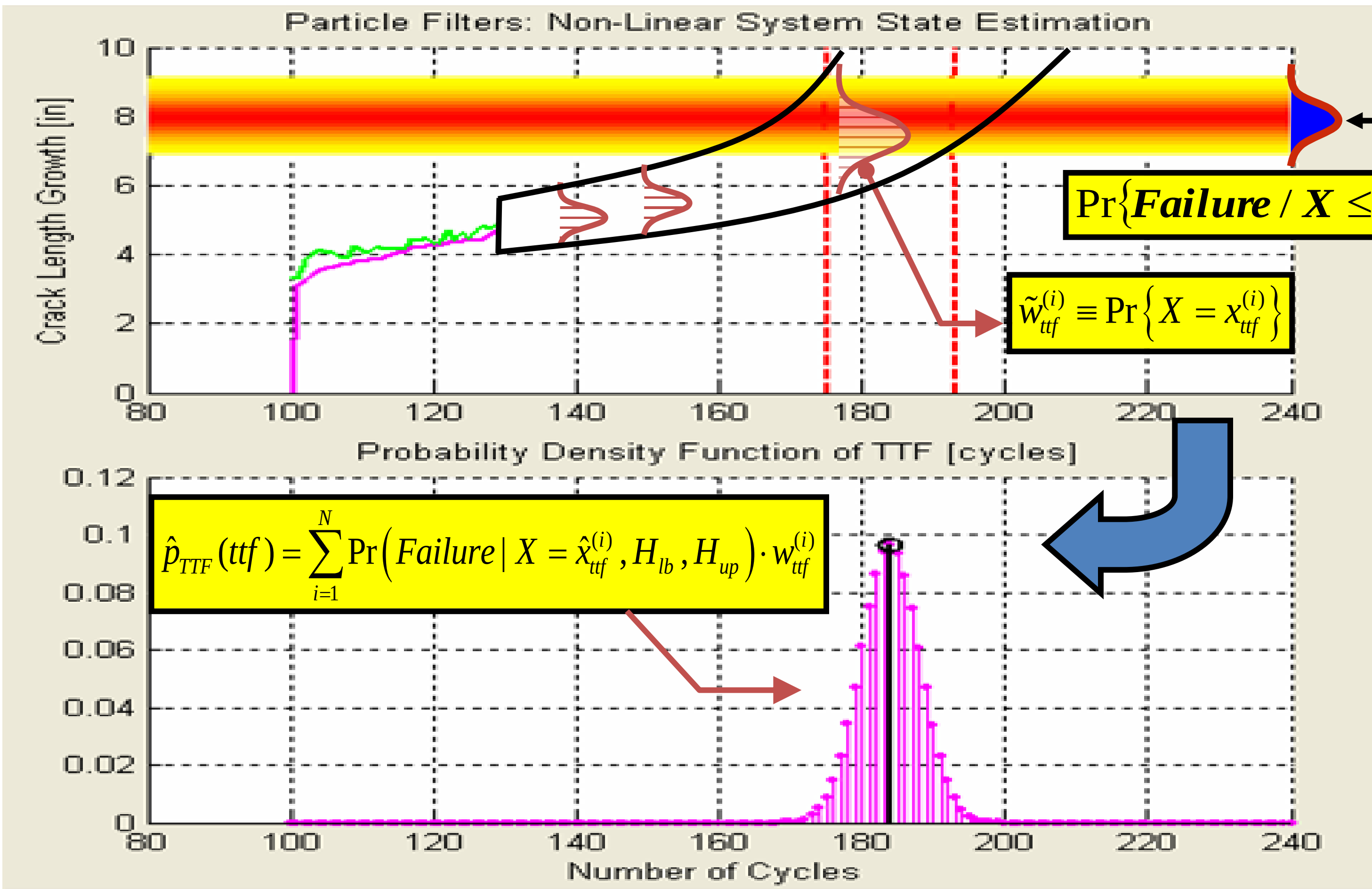
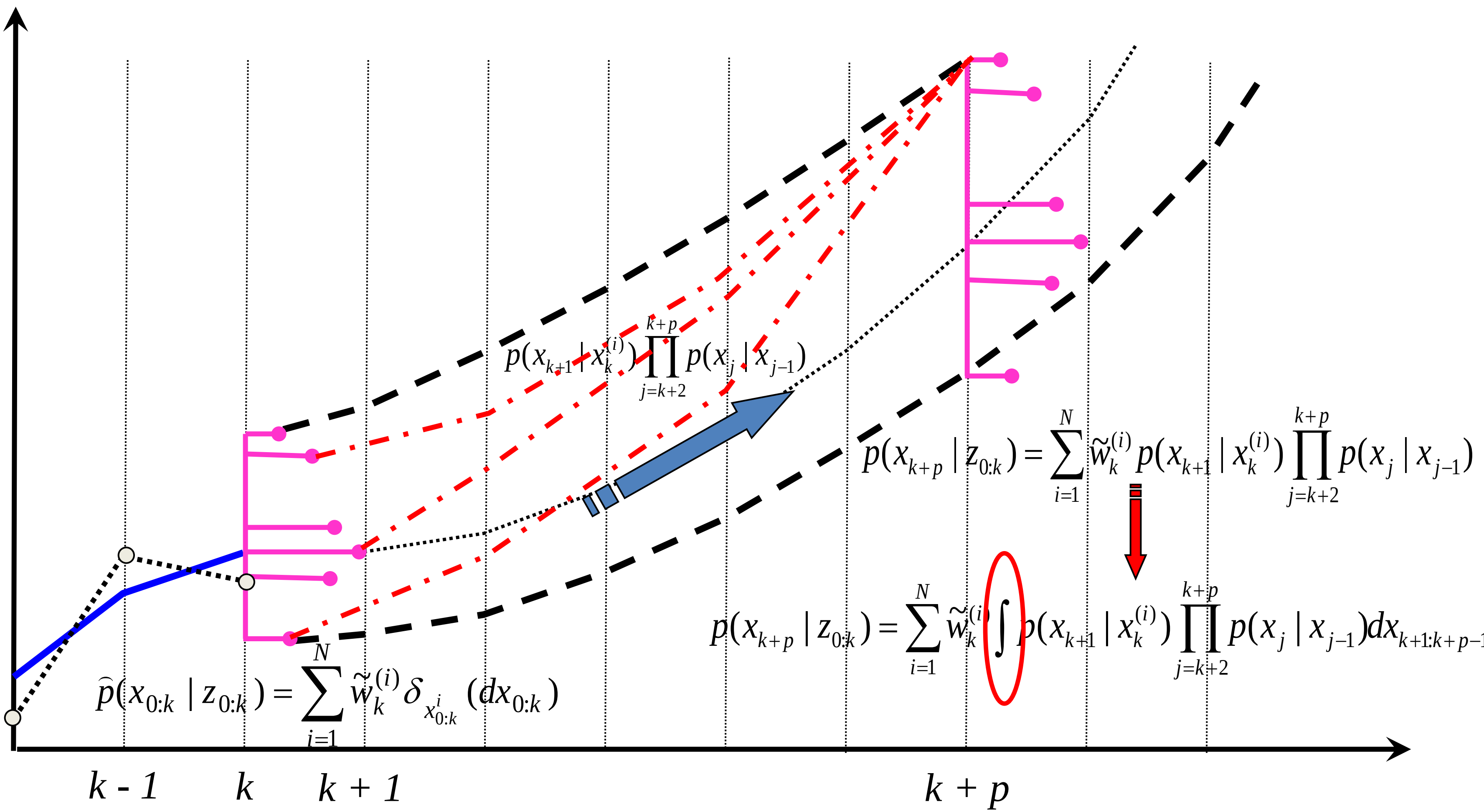
3) PF-based Failure Prognosis



3) PF-based Failure Prognosis



3) PF-based Failure Prognosis

✓ **First Approach for Long-Term Prediction:**
(Weight Update Procedure)

- Predicted Trajectory:

$$\hat{x}_{t+p}^{(i)} = E[f_{t+p}(\tilde{x}_{t+p-1}^{(i)}, \omega_{t+p})] \quad ; \quad \hat{x}_t^{(i)} = \tilde{x}_t^{(i)}$$

- Predicted State pdf @ time *t+k*

$$\hat{p}(x_{t+k} \mid \hat{x}_{1:t+k-1}) \approx \sum_{i=1}^{N} w_{t+k-1}^{(i)} \cdot \hat{p}(x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)}) \; ; \; k = 1, \cdots, p$$

Predicted Conditional pdf (noise model)

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3) PF-based Failure Prognosis

✓ **First Approach for Long-Term Prediction:**
(Weight Update Procedure)

Weight update for Long-Term Prediction

- Construct a partition of the random variable domain by defining:

$$d_{t+k}^{(1)} = -\infty; \quad d_{t+k}^{(N+1)} = \infty$$

$$d_{t+k}^{(j)} = \frac{1}{2}\left(\hat{x}_{t+k}^{(j)} + \hat{x}_{t+k}^{(j-1)}\right), \quad j = 2, \ldots, N$$

- Generate the updated particle weights by computing:

$$w_{t+k}^{(i)} = \int_{d_{t+k}^{(i)}}^{d_{t+k}^{(i+1)}} \hat{p}(x_{t+k} \mid \hat{x}_{0:t+k-1}, y_{1:t}) dx_{t+k}$$

3) PF-based Failure Prognosis

✓ **Second Approach for Long-Term Prediction: (Regularization of Predicted State pdf)**

- Uncertainty: Resampling procedure for predicted state pdf
- Statistical information given by the position of the particles, not by the particle weight.
- Use of Epanechnikov kernels

$$K_{opt}(x) = \begin{cases} \frac{n_x + 2}{2c_{n_x}}\left(1 - \|x\|^2\right) & \text{if } \|x\| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{p}(x_{t+k} \mid \hat{x}_{1:t+k-1}) \approx \sum_{i=1}^{N} w_{t+k-1}^{(i)} K\left(x_{t+k} - E\left[x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)}\right]\right)$$

$$\int \|x\|^2 K(x)\,dx < \infty \qquad \int xK(x)\,dx$$

3) PF-based Failure Prognosis

✓ **Second Approach for Long-Term Prediction:**
(Regularization of Predicted State pdf)

Long Term Predictions: Second Approach

- For $i = 1, \cdots, N$, $w_{t+k}^{(i)} = N^{-1}$
- Calculate $\hat{S}_{t+k}$, the empirical covariance matrix of $\left\{ E\left[x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)} \right], w_{t+k}^{(i)} \right\}_{i=1}^{N}$
- Compute $\hat{D}_{t+k}$ such that $\hat{D}_{t+k} \hat{D}_{t+k}^{T} = \hat{S}_{t+k}$
- For $i = 1, \cdots, N$, draw $\varepsilon^{i} \sim K$, the Epanechnikov kernel and assign

$$\hat{x}_{t+k}^{(i)*} = \hat{x}_{t+k}^{(i)} + h_{t+k}^{opt} \hat{D}_{t+k} \varepsilon^{i}$$

$$K_{opt}(x) = \begin{cases} \dfrac{n_x + 2}{2 c_{n_x}} \left(1 - \|x\|^2 \right) & \text{if } \|x\| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$h_{opt} = A \cdot N^{-\frac{1}{n_x + 4}}$$

$$A = \left(8 c_{n_x}^{-1} \cdot (n_x + 4) \cdot \left(2\sqrt{\pi} \right)^{n_x} \right)^{\frac{1}{n_x + 4}}$$

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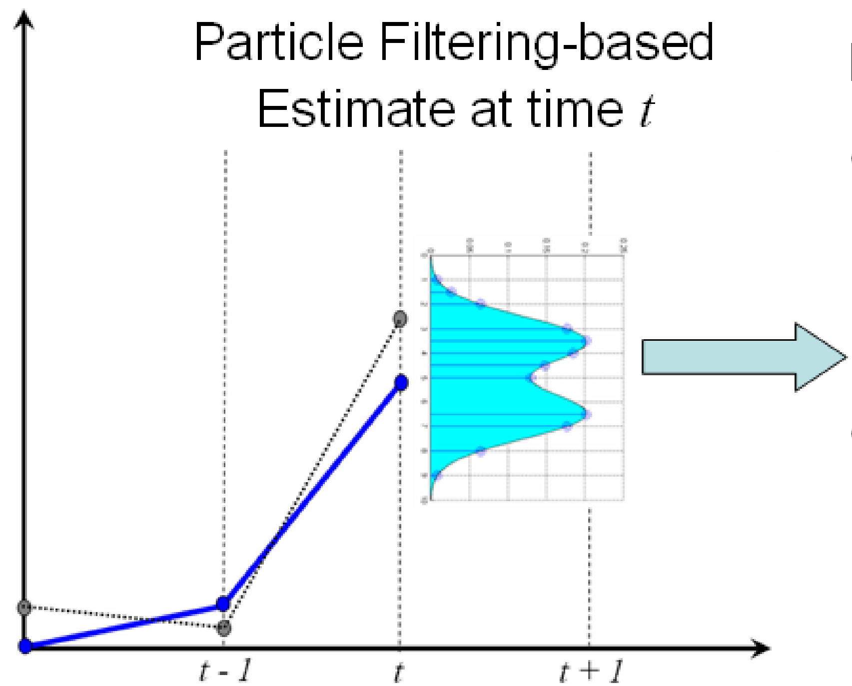
3) PF-based Failure Prognosis



For k = 1, 2, 3, …

- Use nonlinear State equation and Inverse Transform Resampling to obtain a set of equally weighted particles centered at $\left\{E\left[x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)}\right]\right\}_{i=1}^{N}$
- Use Epanechnikov kernels and the Regularization algorithm to obtain a new set of equally weighted particles $\left\{\hat{x}_{t+k}^{(i)}\right\}_{i=1}^{N}$

Long Term Predictions: Regularization of Predicted State PDF

- Apply modified inverse transform resampling procedure. For $i = 1, \cdots, N$, $w_{t+k}^{(i)} = N^{-1}$
- Calculate $\hat{S}_{t+k}$, the empirical covariance matrix of $\left\{E\left[x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)}\right], w_{t+k}^{(i)}\right\}_{i=1}^{N}$
- Compute $\hat{D}_{t+k}$ such that $\hat{D}_{t+k}\hat{D}_{t+k}^{T} = \hat{S}_{t+k}$
- For $i = 1, \cdots, N$, draw $\varepsilon^{i} \sim K$, an Epanechnikov kernel and assign $\hat{x}_{t+k}^{(i)*} = \hat{x}_{t+k}^{(i)} + h_{t+k}^{opt}\hat{D}_{t+k}\varepsilon^{i}$

3) PF-based Failure Prognosis

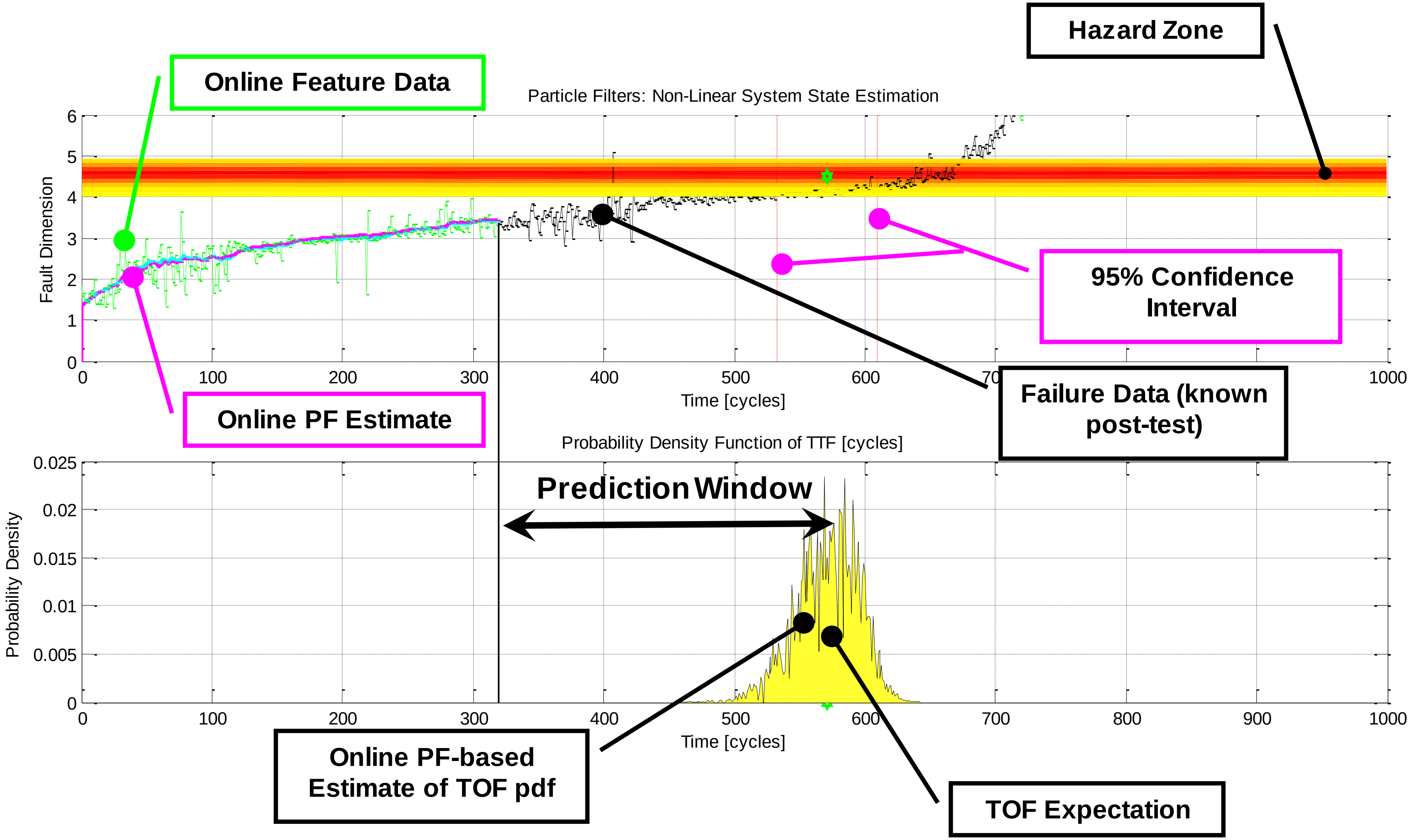
✓ **Third Approach for Long-Term Prediction**: (**Projection in Time of State Expectations**)

$$\hat{x}_{t+p}^{(i)} = E[f_{t+p}(\tilde{x}_{t+p-1}^{(i)}, \omega_{t+p})] \quad ; \quad \hat{x}_t^{(i)} = \tilde{x}_t^{(i)}$$

$$w_{t+k}^{(i)} = w_{t+k-1}^{(i)} \; ; \; k = 1, \cdots, p$$

- Simpler in terms of computational effort.
- Particle weights invariant for future time instants.
- When it works, sources of error are negligible compared to:
  - model inaccuracies
  - wrong assumptions about noise parameters

3) PF-based Failure Prognosis



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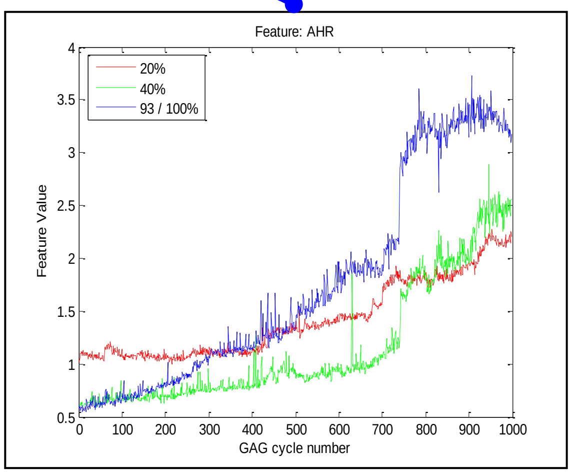
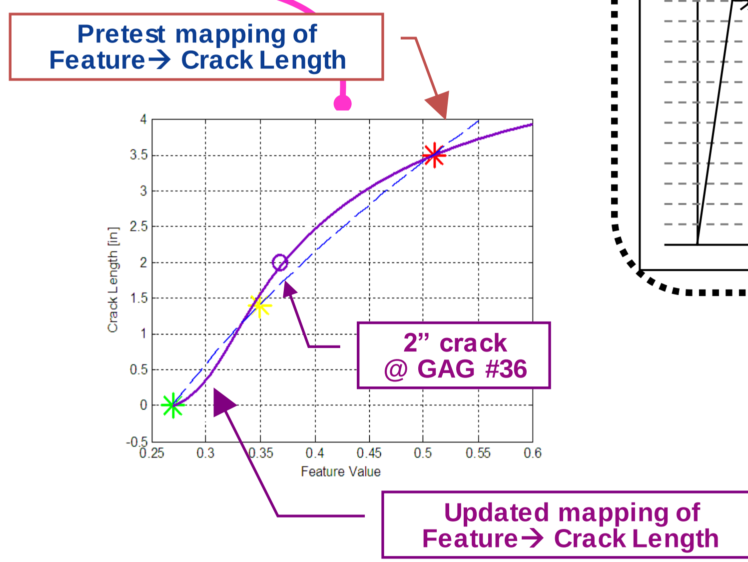
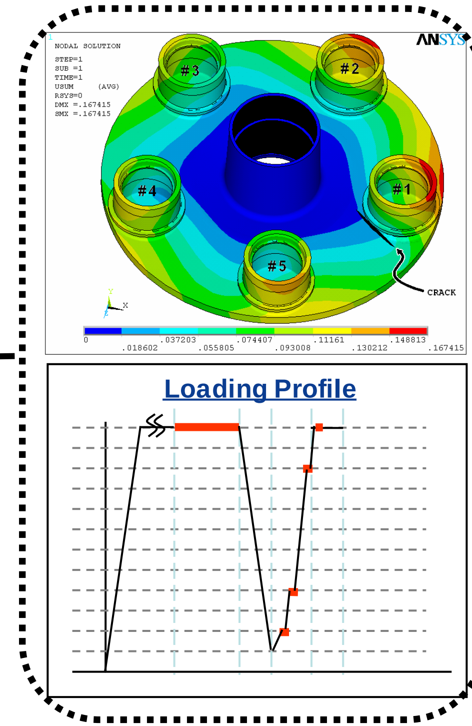
3) PF-based Failure Prognosis

$$
\begin{cases}
L(t+1) = L(t) + C \cdot \alpha(t) \cdot \left\{ \left(\Delta K_{inboard}(t) \right)^m + \left(\Delta K_{outboard}(t) \right)^m \right\} + \omega_1(t) \\
\alpha(t+1) = \alpha(t) + \omega_2(t) \\
\Delta K_{inboard}(t) = f_{inboard}\left(\text{Load}(t), L(t) \right) \\
\Delta K_{outboard}(t) = f_{outboard}\left(\text{Load}(t), L(t) \right)
\end{cases}
$$

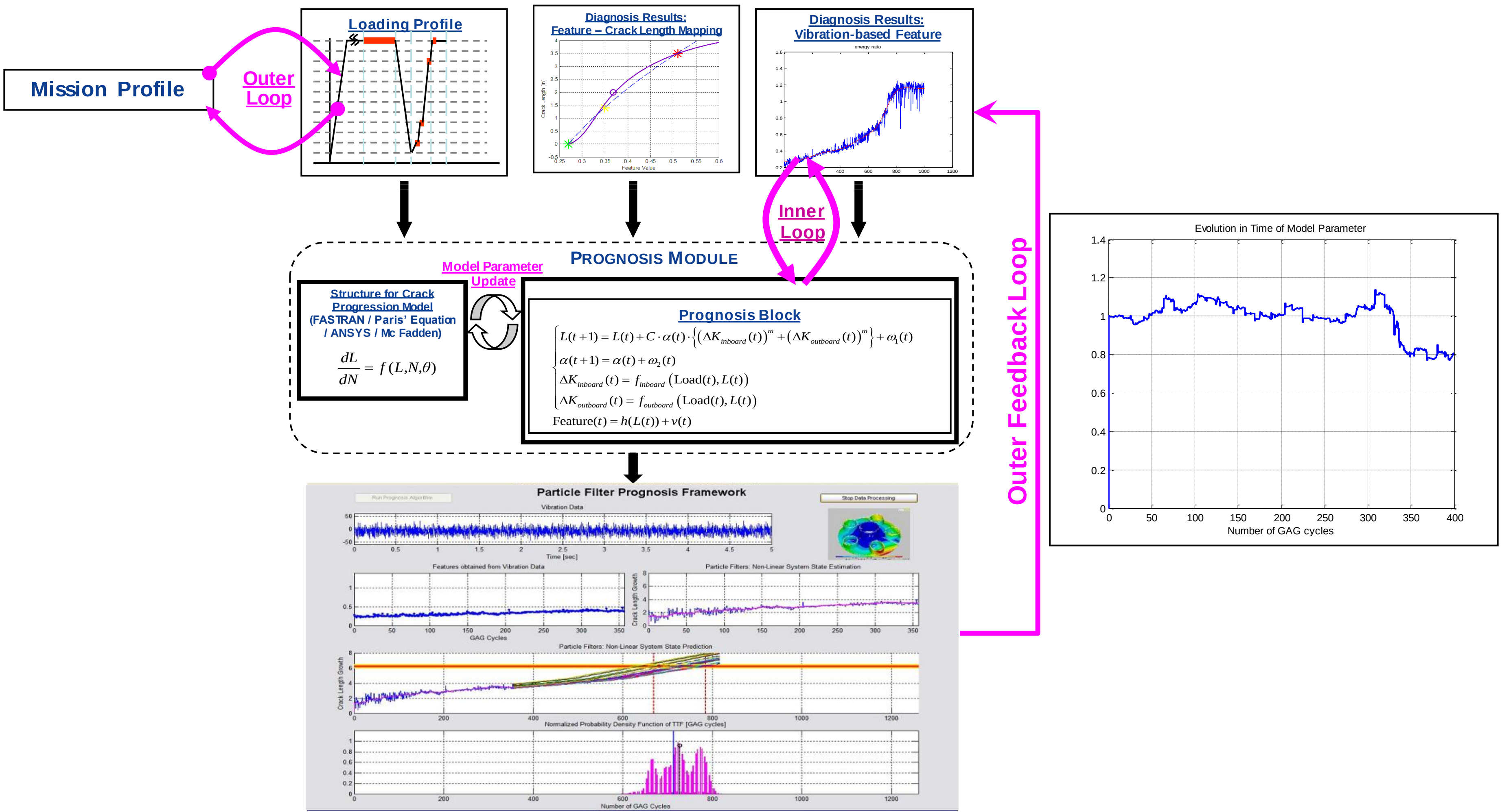
$$
\text{Feature}(t) = h(L(t)) + v(t)
$$



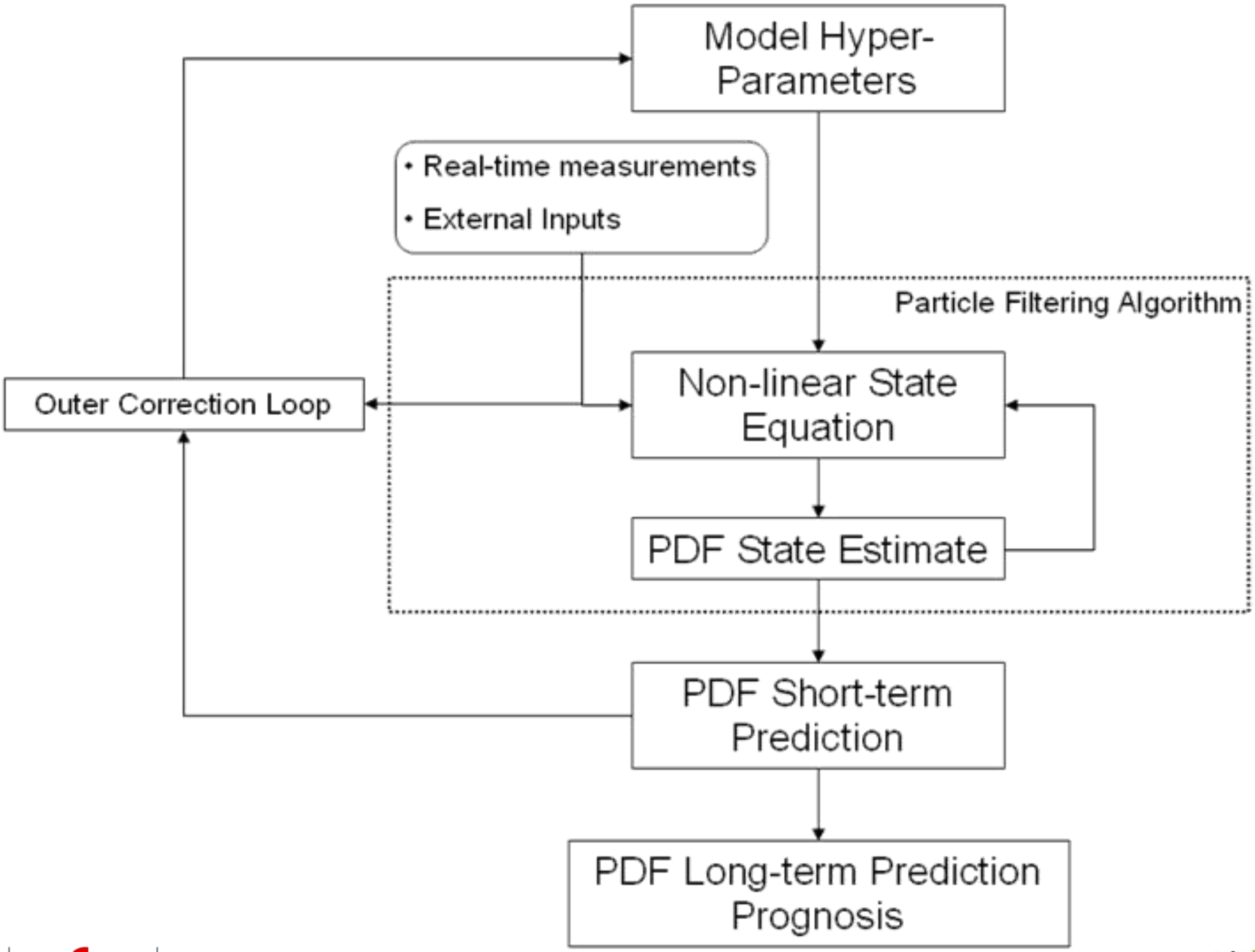




4) Parameter Uncertainty and Outer Correction Loops



4) Parameter Uncertainty and Outer Correction Loops



4) Parameter Uncertainty and Outer Correction Loops

- **Concept of "Artificial Evolution" revised**

$$\begin{cases} x(t+1) = f_t(x(t), x_\alpha(t), \omega_1(t)) \\ x_\alpha(t+1) = x_\alpha(t) + \omega_\alpha(t) \\ \text{Features}(t) = h_t(x(t), x_\alpha(t), v(t)) \end{cases}$$

- f_t and h_t are non-linear mappings.
- $x(t)$ is the state vector.
- $\omega_1(t)$ and $v(t)$ are non-Gaussian distributions
- $x_\alpha(t)$ is an state associated with an unknown model parameter α
- $\omega_\alpha(t)$ is zero-mean random noise

4) Parameter Uncertainty and Outer Correction Loops

- **Proposed Outer Correction Loop:**

$$\begin{cases} \mathrm{var}\{\omega_\alpha(t+1)\} = p\cdot\mathrm{var}\{\omega_\alpha(t)\}, \text{ if } \dfrac{\|Pred\_error(t)\|}{\|Feature(t)\|} < Th \\ \mathrm{var}\{\omega_\alpha(t+1)\} = q\cdot\mathrm{var}\{\omega_\alpha(t)\}, \text{ if } \dfrac{\|Pred\_error(t)\|}{\|Feature(t)\|} > Th \end{cases}$$

- $0 < p < 1$, $q > 1$, and $0 < Th < 1$ are scalars

4) Parameter Uncertainty and Outer Correction Loops

- **Formally speaking…**

- Assume a nonlinear state equation: $\begin{cases} x_{k+1} = x_k + \alpha_k \cdot F(x_k, \alpha_k) + \omega_k \\ \alpha_{k+1} = L(\alpha_k, e_k^s) + \omega_k{}' \end{cases}$

  where $L(\alpha_k, e_k^s) = \alpha_k$ $\qquad y_k = x_k + \upsilon_k$

- **<u>First Approach</u>:**

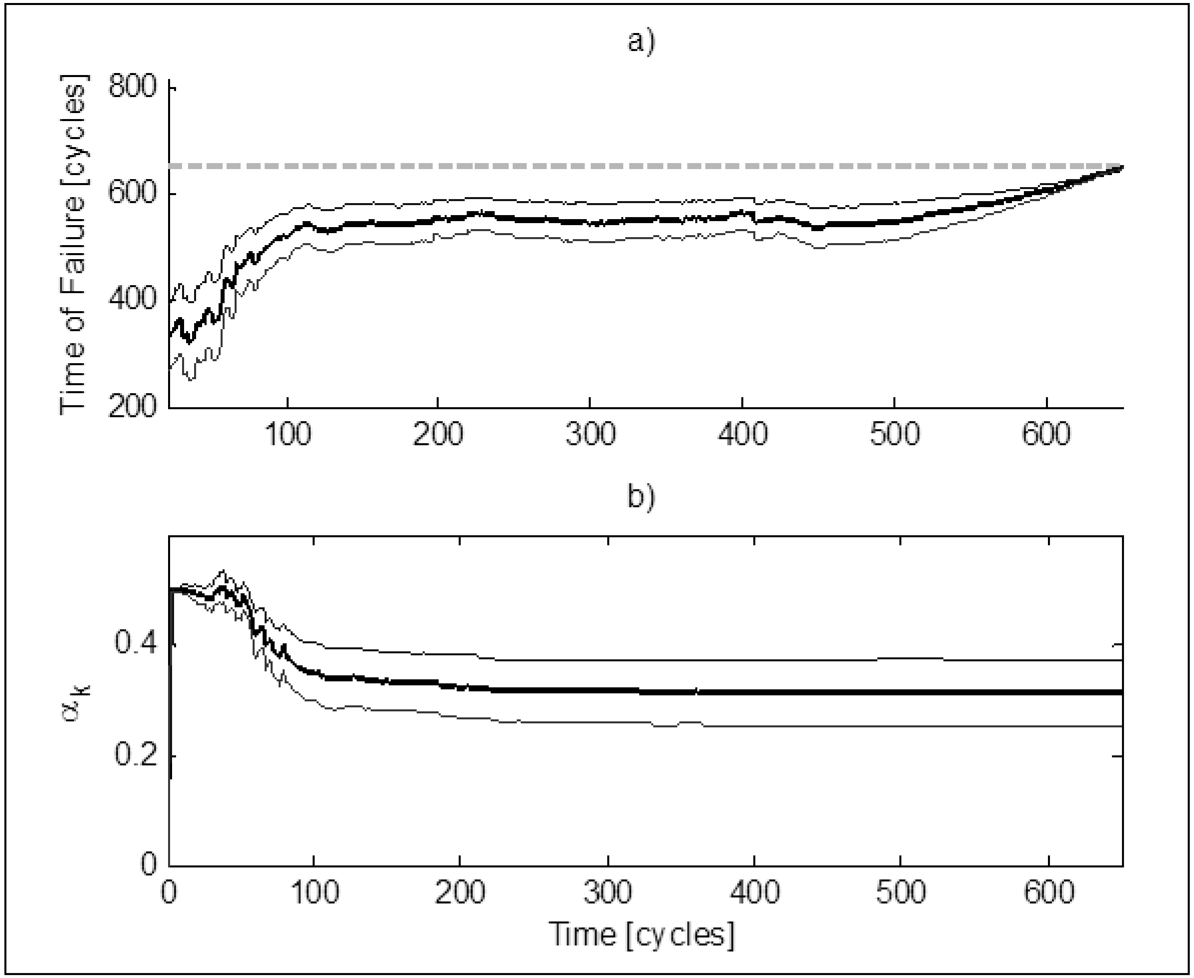
$$var(\omega_k') := \begin{cases} p \cdot var(\omega_k') & \left|e_k^s\right| \le e^{th} \\ q \cdot var(\omega_k') & \left|e_k^s\right| > e^{th} \end{cases}$$

- **<u>Second Approach</u>:**

$$L(\alpha_k, e_k^s) := \begin{cases} \alpha_k & \left|e_k^s\right| \le e^{th} \\ \alpha_k + \eta e_k^s & \left|e_k^s\right| > e^{th} \end{cases}, \quad var(\omega_{k+1}') := \begin{cases} p \cdot var(\omega_k') & \left|e_k^s\right| \le e^{th} \\ \sigma_0^2 & \left|e_k^s\right| > e^{th} \end{cases}$$

4) Parameter Uncertainty and Outer Correction Loops

- **Classic PF-based Prognosis Framework:**



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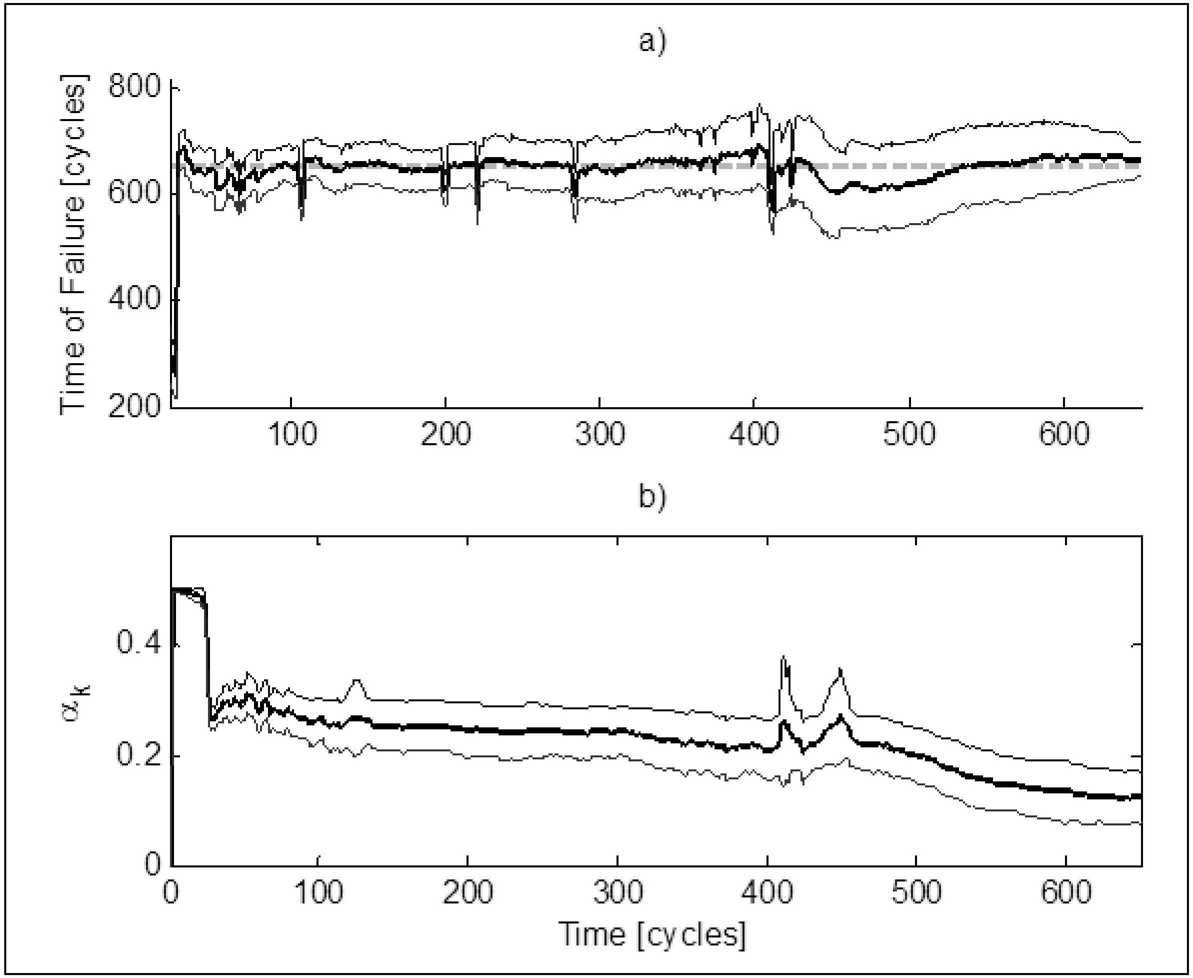
4) Parameter Uncertainty and Outer Correction Loops

- **Outer Correction Loops in a PF-based Prognosis Framework:**



4) Parameter Uncertainty and Outer Correction Loops

- **Results for Outer Correction Loops in a case study (several runs of the algorithm, given the stochastic nature of the filtering algorithm)**

✓ Outer Correction Loop that modifies only the variance of model hyper-parameters:

Mean of ToF Expectation = 540 cycles (ground truth = 650 cycles)
Mean of 95% CI Lower Limit = 503 cycles
Mean of 95% CI Upper Limit = 573 cycles

✓ Outer Correction Loop that modifies only the expectation and variance of hyper-parameters:

Mean of ToF Expectation = 645 cycles (ground truth = 650 cycles)
Mean of 95% CI Lower Limit = 608 cycles
Mean of 95% CI Upper Limit = 681 cycles

5) Performance Measures for Prognostic Algorithms

➢ RUL On-line Precision Index (RUL-OPI):

- Considers the relative length of the 95% confidence interval computed at time t (CI_t), when compared to the remaining useful life.
- Quantifies the concept: “the more data the algorithm processes, the more precise the prognostic result”
- Good prognostic results are associated to values of $I_1(t) \approx 1$

$$I_1(t) = e^{-\left(\frac{\sup(CI_t) - \inf(CI_t)}{E_t\{RUL\}}\right)} = e^{-\left(\frac{\sup(CI_t) - \inf(CI_t)}{E_t\{ToF\} - t}\right)}$$

$$0 < I_1(t) \leq 1, \forall t \in [1, E_t\{ToF\}), t \in \square$$

5) Performance Measures for Prognostic Algorithms

➢ RUL Accuracy-Precision Index:

- Considers the error in the ToF estimate with respect to the length of the 95% confidence interval computed at time t (Ci_t) and penalizes the fact that $E_t\{ToF\} > Ground\,Truth\{ToF\}$

- Good prognostic results are associated to values of the index such that $0 \leq 1 - I_2(t) \leq \varepsilon$

  where ε is a small positive constant

$$I_2(t) = e^{-\left(\frac{Ground\,Truth\{ToF\} - E_t\{ToF\}}{\sup(CI_t) - \inf(CI_t)}\right)}$$

$$0 < I_2(t), \forall t \in [1, E_t\{ToF\}), t \in \mathbb{N}$$

5) Performance Measures for Prognostic Algorithms

➢ RUL On-line Steadiness Index (RUL-OSI):

- Considers the current estimate for the expectation of the time of failure (ToF) computed at time t.
- Quantifies the concept: “the more data the algorithm processes, the more steady the prognostic result”
- Good prognostic results are associated to small values for the RUL-OSI

$$I_3(t) = \sqrt{Var\left(E_t\left\{ToF\right\}\right)}$$

$$I_3(t) \geq 0, \forall t \in \mathbb{R}$$

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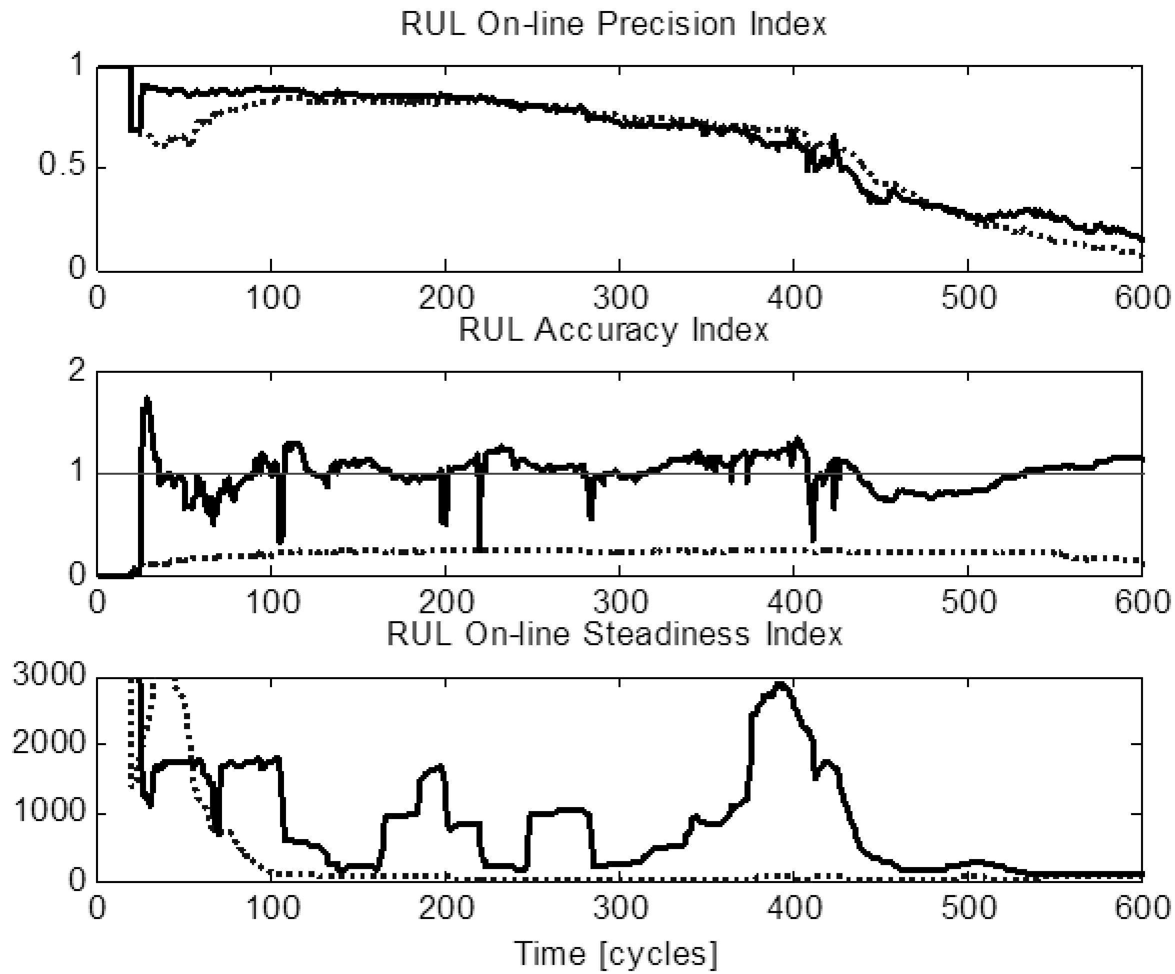
5) Performance Measures for Prognostic Algorithms

- **Application examples…**

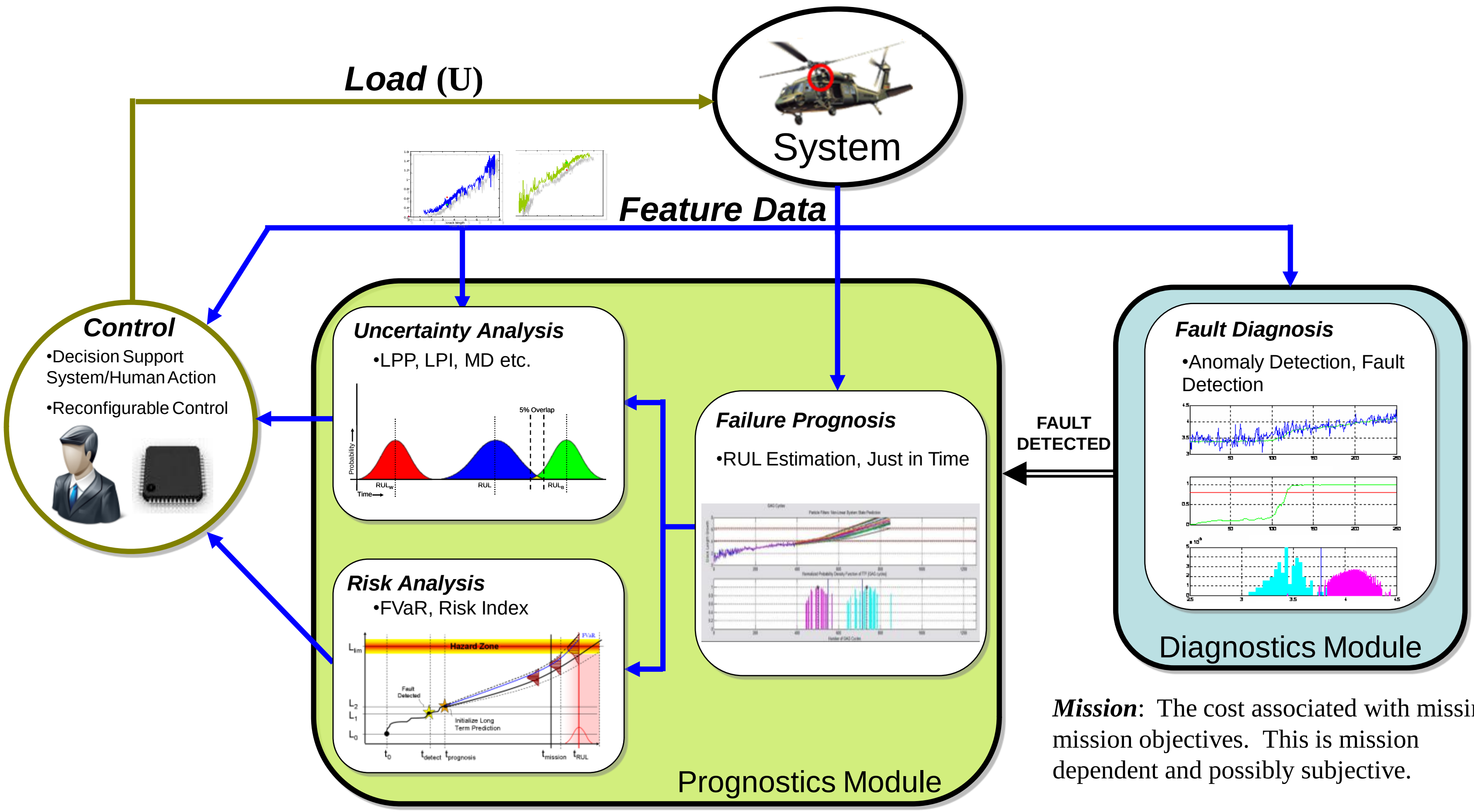


6) Input Uncertainty in PF-based Prognostic Algorithms

- In order to accurately predict the Remaining Useful Life (RUL) of a failing system, one must consider the future, and often unpredictable, stresses that will be acting on the system.
  - How do these stresses affect the Remaining Useful Life (RUL)?
  - How does uncertainty in these stresses affect the RUL estimate?
  - How can uncertainty be quantified?

- Only after addressing these issues, it is possible to answer one particularly interesting question:
  - How can knowledge of uncertainty be used to extend the RUL of a failing system?

6) Input Uncertainty in PF-based Prognostic Algorithms



Mission: The cost associated with missing mission objectives. This is mission dependent and possibly subjective.

6) Input Uncertainty in PF-based Prognostic Algorithms

- A number of elements can alter in a significant manner the RUL of equipment and components.
- Consider, for example, uncertainty associated to load profiles, model errors, and measurement noise.
- Thus, RUL uncertainty (ΔRUL) can be written as:
- Level 1: $$\Delta RUL = \left\{ \left[\frac{\partial RUL}{\partial model} \Delta model \right]^2 + \left[\frac{\partial RUL}{\partial load} \Delta load \right]^2 + \left[\frac{\partial RUL}{\partial meas.} \Delta meas. \right]^2 \right\}^{1/2}$$
- Level 2: $$\Delta load = \left\{ \left[\frac{\partial load}{\partial \text{mission}} \Delta \text{mission} \right]^2 + \left[\frac{\partial load}{\partial \text{regime data}} \Delta \text{regime data} \right]^2 + \left[\frac{\partial load}{\partial \text{sensors}} \Delta \text{sensors} \right]^2 \right\}^{1/2}$$
- Level 3: This reasoning can be extrapolated analogously…

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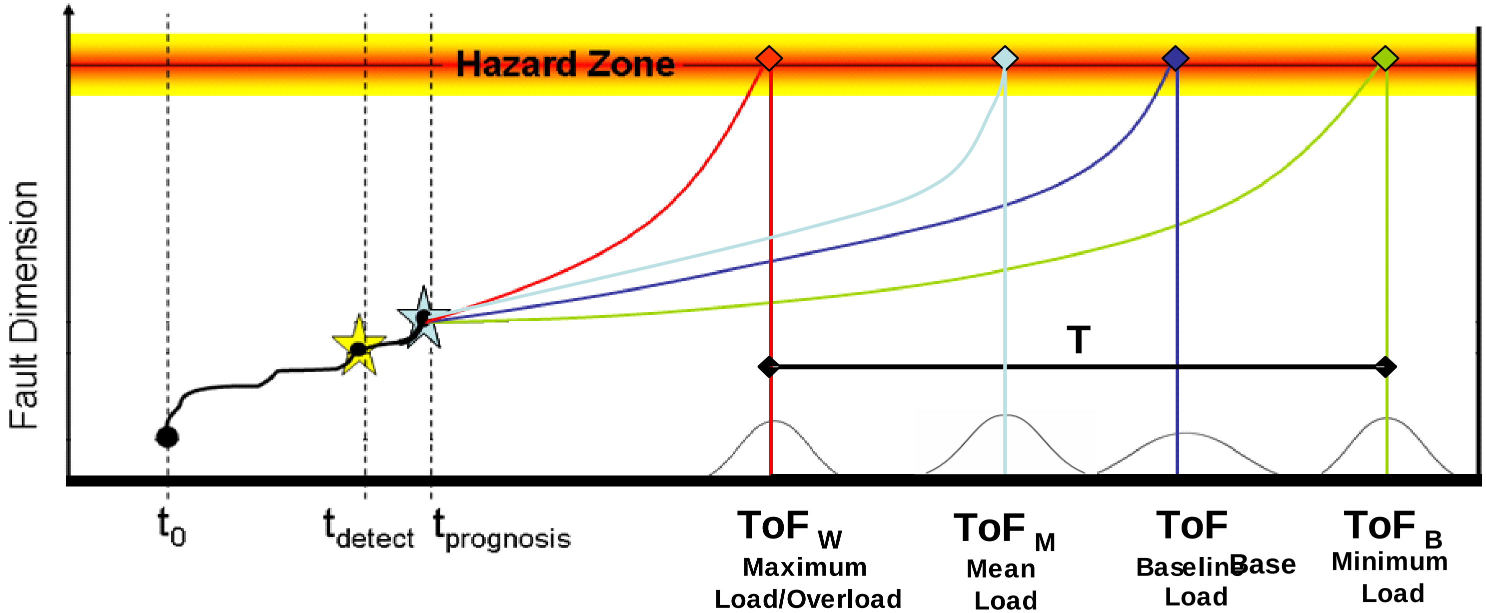
6) Input Uncertainty in PF-based Prognostic Algorithms

- Particle Filter (PF) algorithms have become a key component of failure prognosis frameworks:
  - Strong mathematical foundation
  - Allow online uncertainty representation of state estimates and long-term predictions in nonlinear systems
  - Allow online uncertainty management via the implementation of outer feedback correction loops.

- These facts motivate the usage of PF-based uncertainty measures to quantify, in real time, the impact of load, environmental, and other stresses for long-term prediction.

6) Input Uncertainty in PF-based Prognostic Algorithms

- If the input of the system is also assumed to be a stochastic process:



- Given $P\{U=u\}=\sum_{j=1}^{N_u}\pi_j\delta(u-u_j)$,

where $\{u_j\}_{j=1}^{N_u}$ is a set of constant load values, then

$$\hat{p}_{ToF}(t)=\sum_{j=1}^{N_u}\pi_j\sum_{i=1}^{N}\Pr\left(Failure \mid X=\hat{x}_t^{(i)}, U=u_j, H_{lb}, H_{ub}\right)\cdot w_t^{(i)}$$

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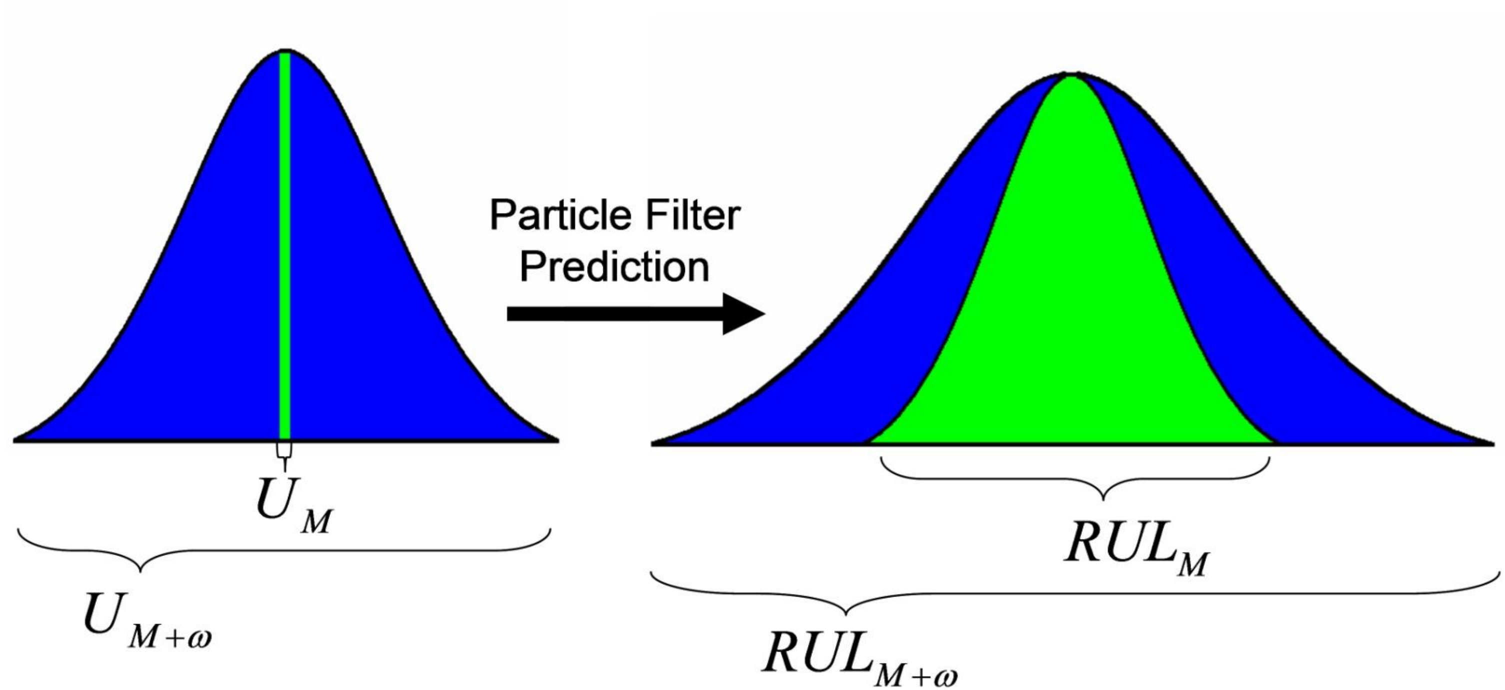
6) Input Uncertainty in PF-based Prognostic Algorithms

- **Dispersion Sensitivity**

$$DS_{\omega} = \frac{stdev(RUL_{Base+\omega})}{stdev(RUL_{Base})}$$

- **Confidence Interval Sensitivity**

$$CIS_{\omega} = \frac{Length(CI\{RUL_{Base+\omega}\})}{Length(CI\{RUL_{Base}\})}$$



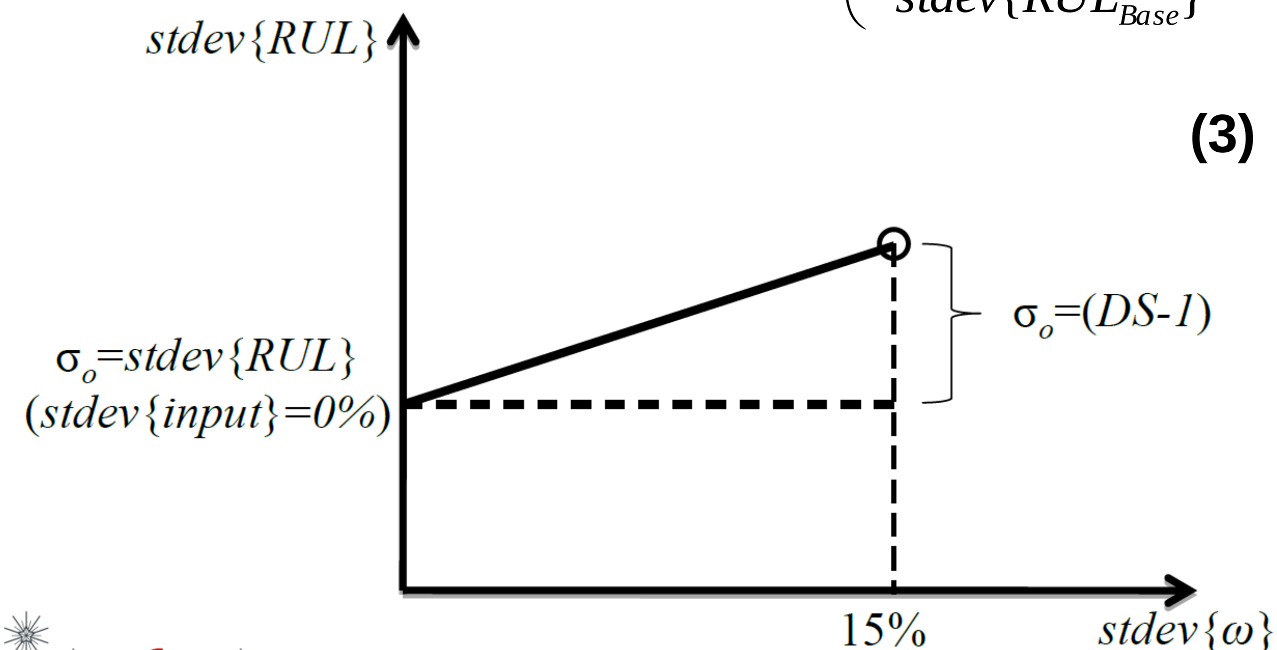
6) Input Uncertainty in PF-based Prognostic Algorithms

- **Dispersion Sensitivity Approach**

(1) $$stdev\{RUL_{Base+\varpi}\} = \frac{RUL_D - E\{RUL_{Base}\}}{Z_{0.95}}$$

(2) $$stdev\{U_{Base+\varpi}\} = \left(\frac{stdev\{RUL_{Base+\varpi}\}}{stdev\{RUL_{Base}\}} - 1\right)\frac{stdev\{\omega\}}{DS-1}$$

(3) $$U_d = U_{Base} - stdev\{U_{Base+\varpi}\}$$



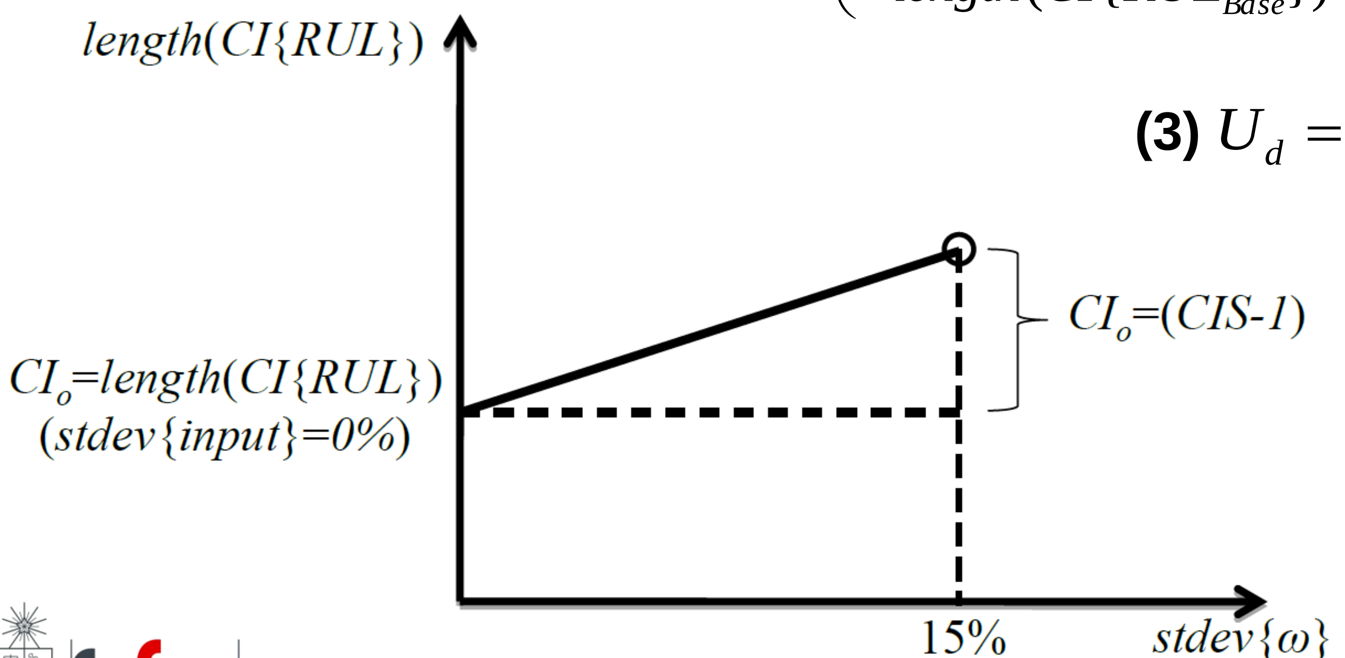
6) Input Uncertainty in PF-based Prognostic Algorithms

- **Confidence Interval Sensitivity Approach**

(1) $$Length(CI\{RUL_{Base+\varpi}\}) = 2(RUL_D - E\{RUL_{Base}\})$$

(2) $$stdev\{U_{Base+\varpi}\} = \left(\frac{Length(CI\{RUL_{Base+\varpi}\})}{length(CI\{RUL_{Base}\})} - 1\right)\frac{stdev\{\omega\}}{CIS - 1}$$

(3) $$U_d = U_{Base} - stdev\{U_{Base+\varpi}\}$$

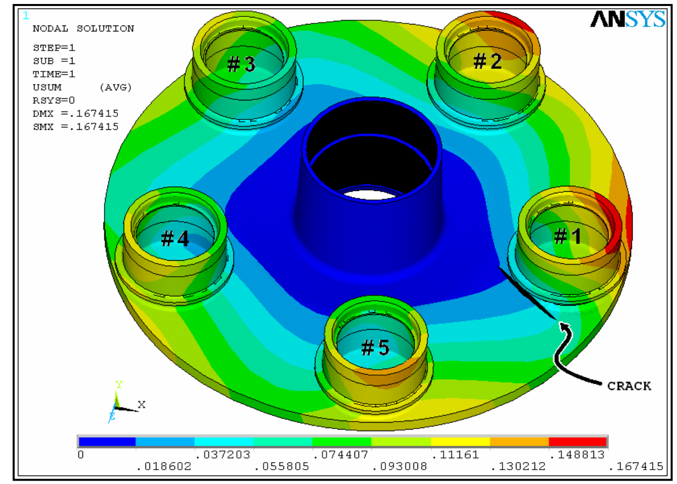


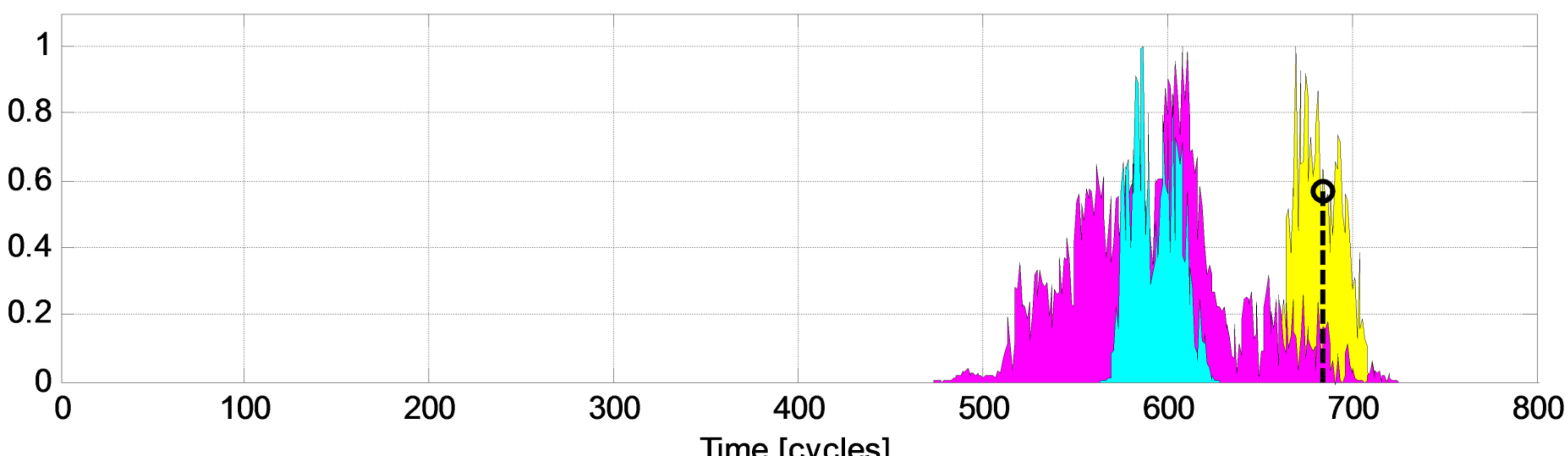
6) Input Uncertainty in PF-based Prognostic Algorithms

Case Study:

A critical component (planetary gear carrier plate) in a rotorcraft transmission system is experiencing a fatigue crack.

The baseline load on the rotorcraft is 120% of the maximum recommended torque. At this load, a failure is predicted to occur at time 594 cycles.





6) Input Uncertainty in PF-based Prognostic Algorithms

- **Dispersion Sensitivity Approach**

U_{Base}=120% ⟹ ToF: 594

U_D=? ⟹ ToF: 714

Dispersion Sensitivity

$$DS_{15\%} = \frac{stdev\{RUL_{Base+\omega}\}}{stdev\{RUL_{Base}\}}$$

$$= \frac{41.52 cycles}{12.44 cycles} = 3.3362$$

6) Input Uncertainty in PF-based Prognostic Algorithms

- **Dispersion Sensitivity Approach**

U_{Base}=120% ⟹ ToF: 594

U_D=? ⟹ ToF: 714

Dispersion Sensitivity

$$DS_{15\%} = \frac{stdev\{RUL_{Base+\omega}\}}{stdev\{RUL_{Base}\}}$$

$$= \frac{41.52 cycles}{12.44 cycles} = 3.3362$$

---

(1) $$stdev\{RUL_{Base+\varpi}\} = \frac{RUL_D - E\{RUL_{Base}\}}{Z_{0.95}} = \frac{714-594}{1.627} = 73.755$$

(2) $$stdev\{U_{Base+\varpi}\} = \left(\frac{stdev\{RUL_{Base+\varpi}\}}{stdev\{RUL_{Base}\}} - 1\right)\frac{stdev\{\omega\}}{DS-1} = 31.64\%$$

(3) $$U_d = U_{Base} - stdev\{U_{Base+\varpi}\} = 120\% - 31.64\% = 88.36\%$$

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6) Input Uncertainty in PF-based Prognostic Algorithms

- **Dispersion Sensitivity Approach**

U_{Base}=120% ⟹ ToF: 594

U_D=88.36% ⟹ ToF: 714

Dispersion Sensitivity

$$DS_{15\%} = \frac{stdev\{RUL_{Base+\omega}\}}{stdev\{RUL_{Base}\}}$$

$$= \frac{41.52 cycles}{12.44 cycles} = 3.3362$$

Actual Results from Fault Testing: U_D=93%

---

(1) $$stdev\{RUL_{Base+\varpi}\} = \frac{RUL_D - E\{RUL_{Base}\}}{Z_{0.95}} = \frac{714-594}{1.627} = 73.755$$

(2) $$stdev\{U_{Base+\varpi}\} = \left(\frac{stdev\{RUL_{Base+\varpi}\}}{stdev\{RUL_{Base}\}} - 1\right)\frac{stdev\{\omega\}}{DS-1} = 31.64\%$$

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6) Input Uncertainty in PF-based Prognostic Algorithms

- **Confidence Interval Sensitivity Approach**

U_{Base}=120% ⟹ ToF: 594

U_D=? ⟹ ToF: 714

Confidence Interval Sensitivity

$$CIS_{15\%} = \frac{Length(CI\{RUL_{Base+\omega}\})}{Length(\{RUL_{Base}\})}$$

$$= \frac{142 cycles}{38 cycles} = 3.7368$$

6) Input Uncertainty in PF-based Prognostic Algorithms

- **Confidence Interval Sensitivity Approach**

Confidence Interval Sensitivity

U_{Base}=120% ⟹ ToF: 594

U_D=? ⟹ ToF: 714

$$CIS_{15\%} = \frac{Length(CI\{RUL_{Base+\omega}\})}{Length(\{RUL_{Base}\})}$$

$$= \frac{142 cycles}{38 cycles} = 3.7368$$

(1) $Length(CI\{RUL_{Base+\varpi}\}) = 2(RUL_D - E\{RUL_{Base}\}) = 2(714-594) = 240$

(2) $stdev\{U_{Base+\varpi}\} = \left(\frac{Length(CI\{RUL_{Base+\varpi}\})}{Length(CI\{RUL_{Base}\})} - 1 \right) \frac{stdev\{\omega\}}{CIS - 1} = 29.13\%$

(3) $U_d = U_{Base} - stdev\{U_{Base+\varpi}\} = 120\% - 29.13\% = 90.87\%$

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6) Input Uncertainty in PF-based Prognostic Algorithms

- **Confidence Interval Sensitivity Approach**

U_{Base}=120% ⟹ ToF: 594

U_D=90.87% ⟹ ToF: 714

Actual Results from Fault Testing: U_D=93%

Confidence Interval Sensitivity

$$CIS_{15\%} = \frac{Length(CI\{RUL_{Base+\omega}\})}{Length(\{RUL_{Base}\})} = \frac{142cycles}{38cycles} = 3.7368$$

---

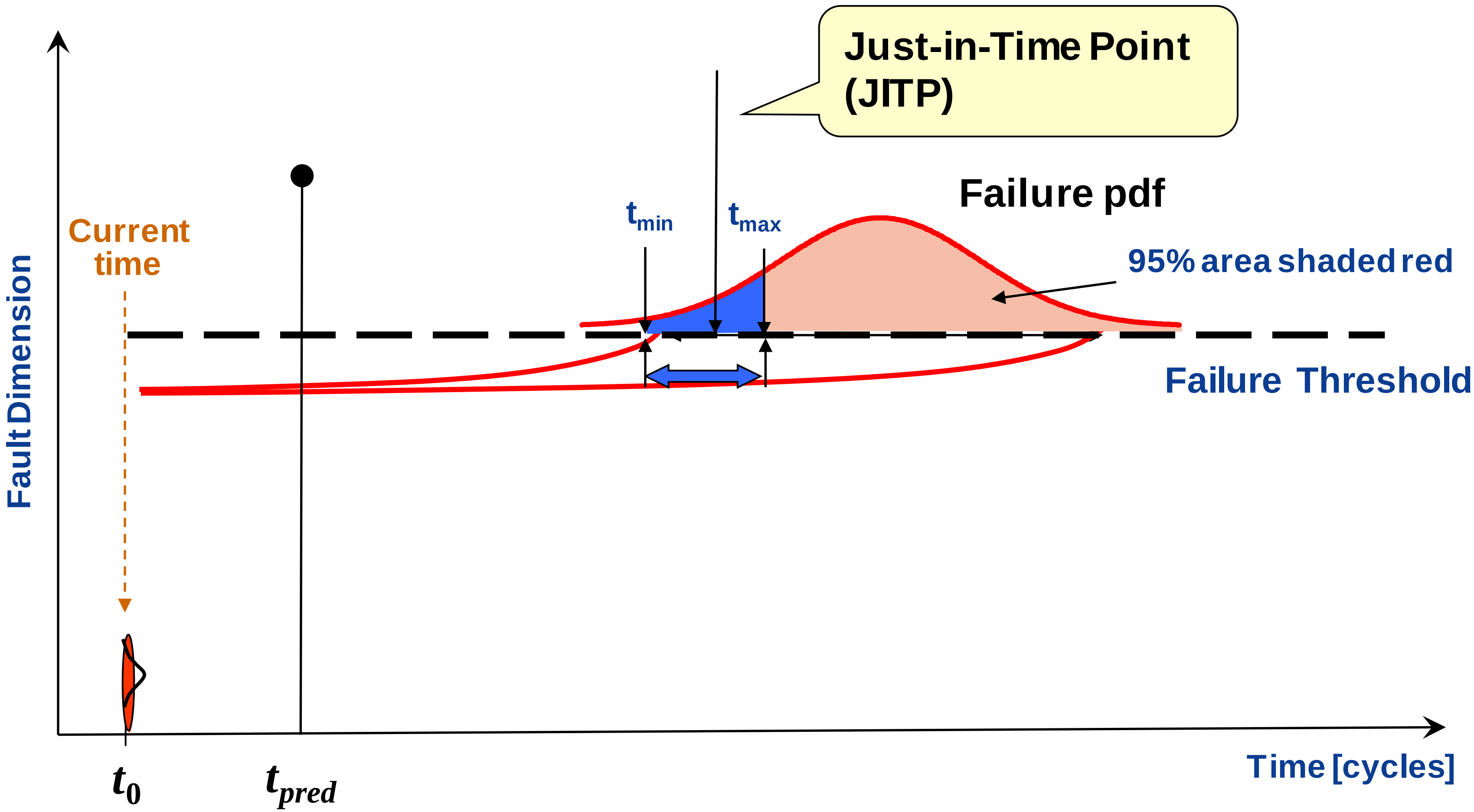
(1) $Length(CI\{RUL_{Base+\varpi}\}) = 2(RUL_D - E\{RUL_{Base}\}) = 2(714 - 594) = 240$

(2) $stdev\{U_{Base+\varpi}\} = \left(\frac{Length(CI\{RUL_{Base+\varpi}\})}{Length(CI\{RUL_{Base}\})} - 1 \right) \frac{stdev\{\omega\}}{CIS - 1} = 29.13\%$

(3) $U_d = U_{Base} - stdev\{U_{Base+\varpi}\} = 120\% - 29.13\% = 90.87\%$

7) Risk Measures for PF-based Prognostic Algorithms

- **Just-in-Time Point vs. RUL Expectations**



7) Risk Measures for PF-based Prognostic Algorithms

➢ **Definition:**

(R1) $\mathcal{R}(C) = C$ for all constants C,

(R2) $\mathcal{R}((1-\lambda)X + \lambda X') \leq (1-\lambda)\mathcal{R}(X) + \lambda\mathcal{R}(X')$ for $\lambda \in (0,1)$ ("convexity")

(R3) $\mathcal{R}(X) \leq \mathcal{R}(X')$ when $X \leq X'$ ("monotonicity")

(R4) $\mathcal{R}(X) \leq 0$ when $||X^k - X||_2 \to 0$ with $\mathcal{R}(X^k) \leq 0$ ("closedness")

- It will also be called a coherent measure of risk in the basic sense if it also satisfies

(R5) $\mathcal{R}(\lambda X) = \lambda\mathcal{R}(X)$ for $\lambda > 0$ ("positive homogeneity")

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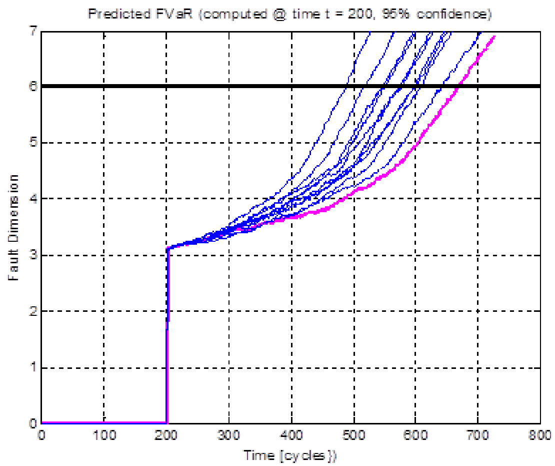
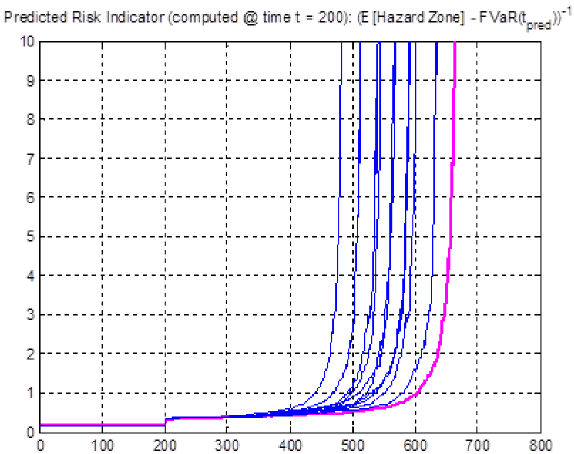
7) Risk Measures for PF-based Prognostic Algorithms

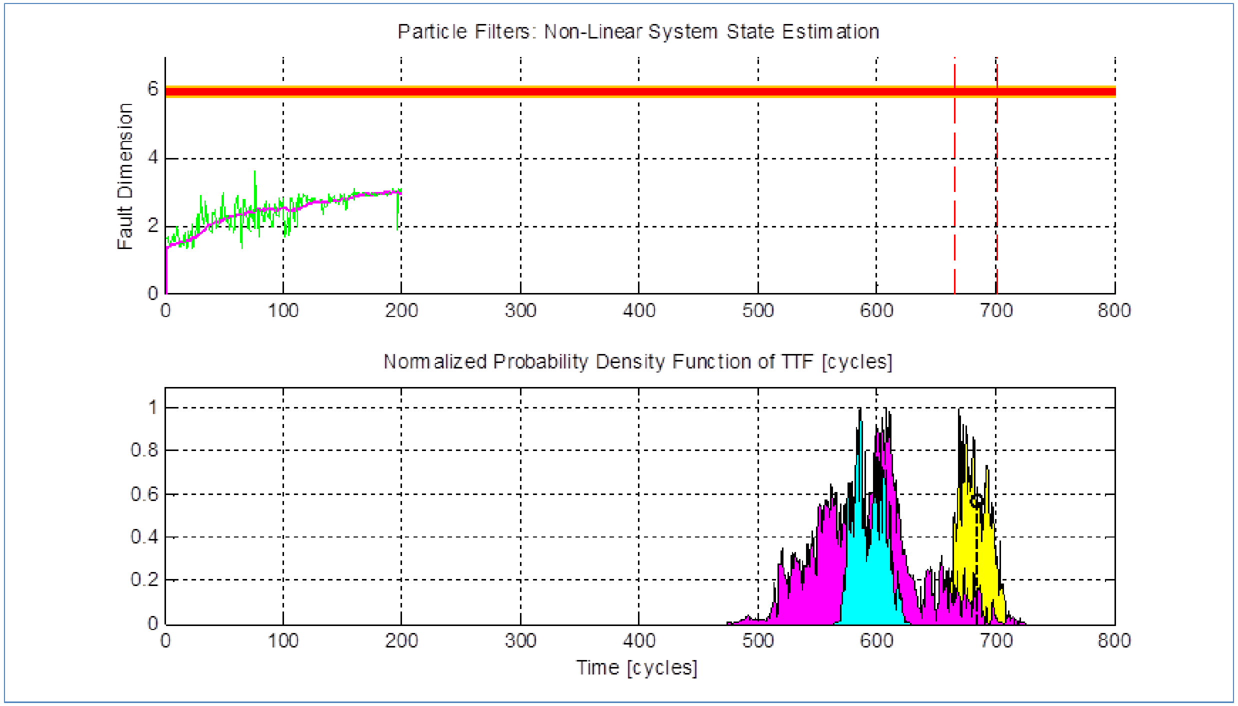
- **Fault Value at Risk (FVaR) and Risk Assessment:**



$$FVaR(t,t_{prognosis}) \Leftrightarrow \alpha = 0.95 = \int_{-\infty}^{FVaR(t,t_{prognosis})} \hat{p}(x_t^1 \mid y_{t_{prognosis}})dx_t^1$$

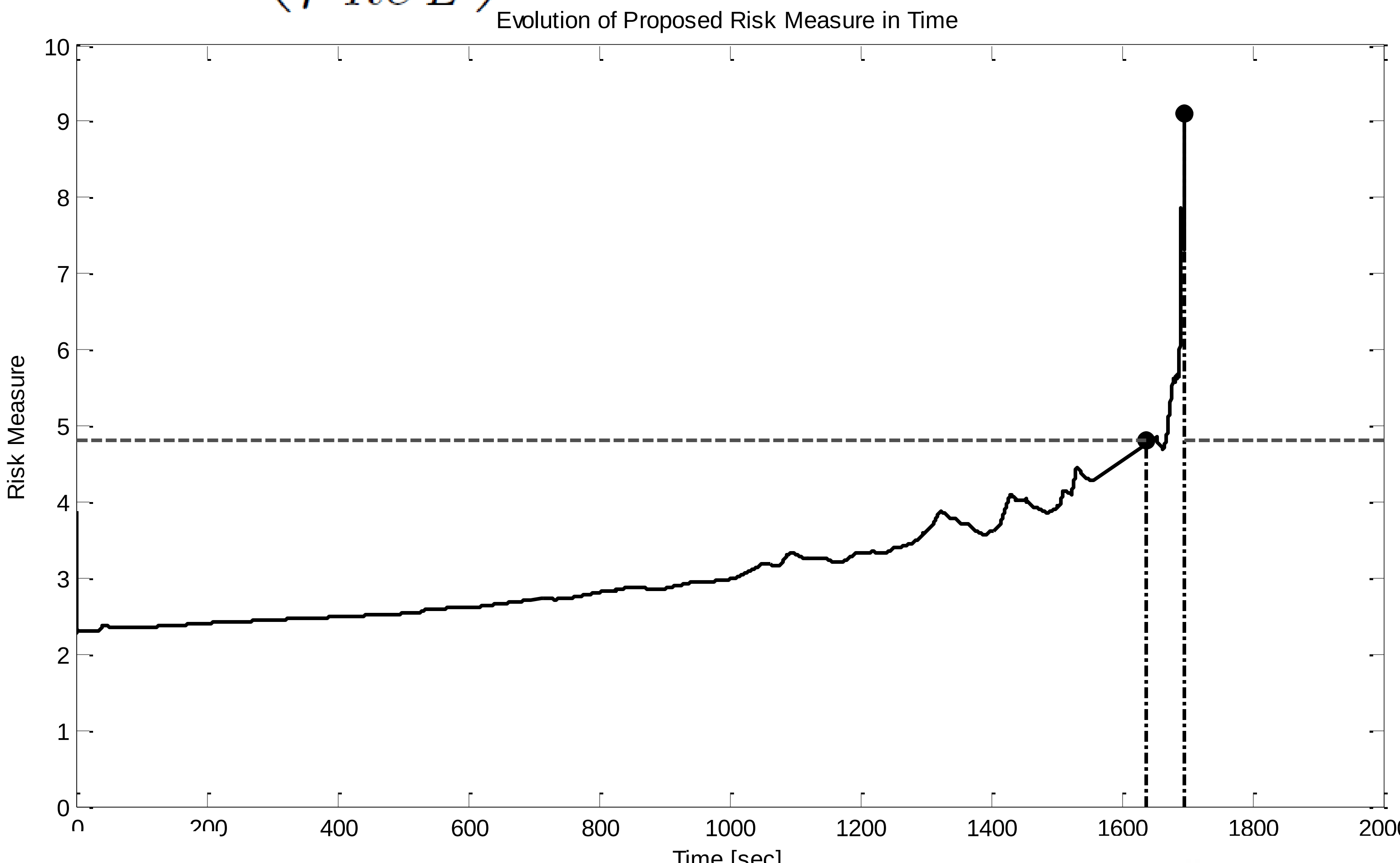
$$Risk_{FVaR}(t,t_{prognosis}) = \left(E\left\{Hazard\ Zone\right\} - FVaR(t,t_{prognosis})\right)^{-1}$$



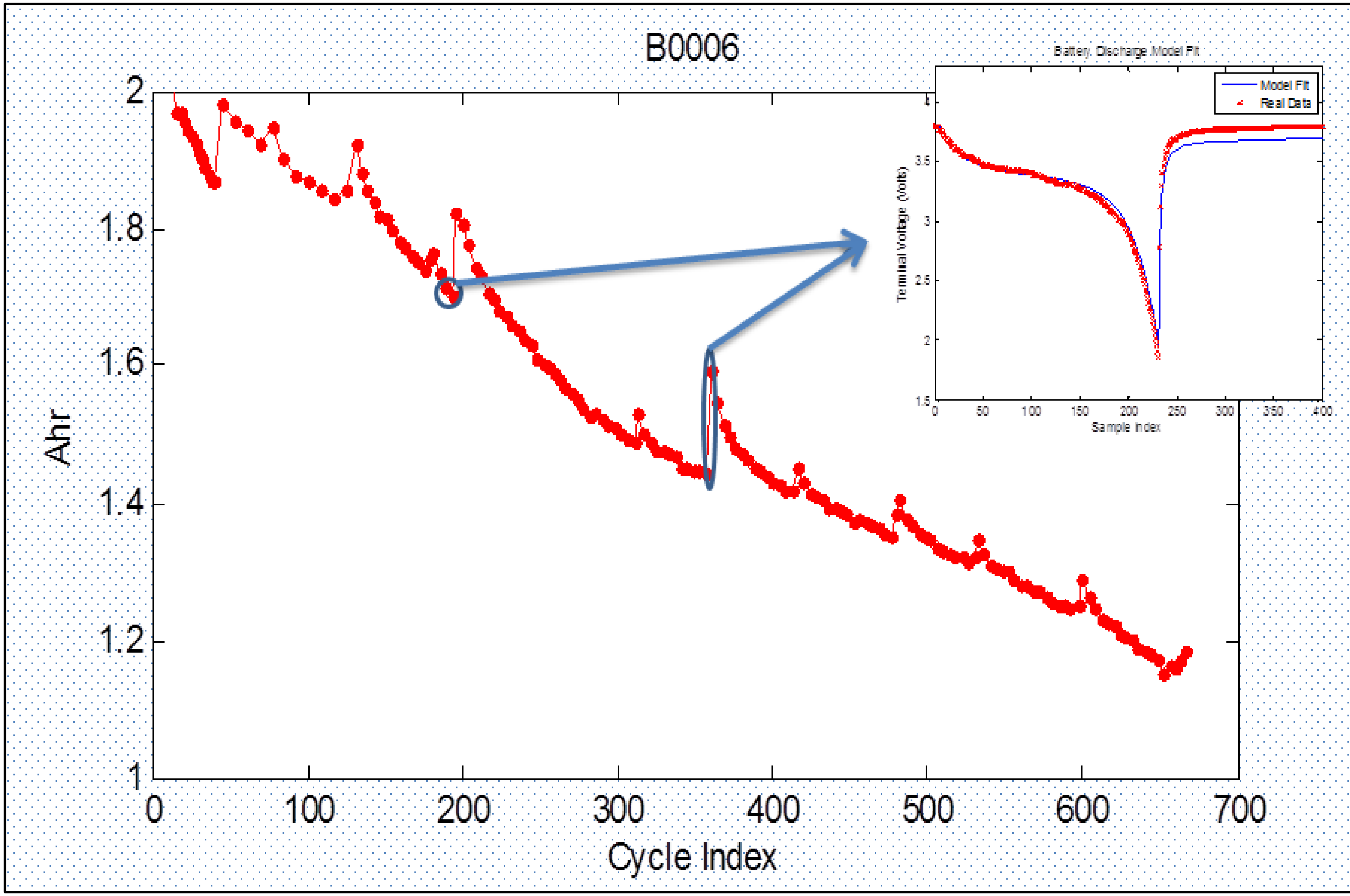


7) Risk Measures for PF-based Prognostic Algorithms

$$\mathcal{R} = \alpha \ln\left(\frac{\beta}{\mu_{RUL}}\right) + \lambda\sigma_{RUL} + \gamma\sigma_I + \delta\sigma_{x_1}(R_N + 1)$$



8.1) Case Study: Battery Diagnostics/Prognostics



8.1) Case Study: Battery Diagnostics/Prognostics

- Data registering two different operational profiles (charge and discharge) at room temperature (NASA Ames Research Center).
- Charging is carried out in a constant current (CC) mode at 1.5[A] until the battery voltage reached 4.2[V] and then continued in a constant voltage mode until the charge current dropped to 20[mA].
- Discharge is carried out at a constant current (CC) level of 2[A] until the battery voltage fell to 2.5[V].
- The experiments were stopped when the batteries reached end-of-life (EOL) criteria, which was a 40% fade in rated capacity (from 2 [A-hr] to 1.2[A-hr]).

8.1) Case Study: Battery Diagnostics/Prognostics

- ***Normal*** condition reflects the fact that the battery SOH is slowly diminishing as a function of the number of charge/discharge cycles

- ***Anomalous*** condition indicates an abrupt increment in the battery SOH (regeneration phenomena).

- To detect the condition of interest, a PF-based anomaly detection module is implemented using nonlinear model

8.1) Case Study: Battery Diagnostics/Prognostics

Anomaly Detection Module: Self-recharge Phenomena

- **State Equation Dynamic Model**

$$\begin{cases} \begin{bmatrix} x_{d,1}(t+1) \\ x_{d,2}(t+1) \end{bmatrix} = f_b\left(\begin{bmatrix} x_{d,1}(t) \\ x_{d,2}(t) \end{bmatrix} + n(t) \right) \\ x_{c1}(t+1) = (1-\beta)x_{c1}(t) + \omega_1(t) \\ x_{c2}(t+1) = 0.95x_{c2}(t)\cdot x_{d,2}(t) + 0.2x_{d,1}(t) + \omega_2(t) \end{cases}$$

$$y(t) = x_{c1}(t) + x_{c2}(t)\cdot x_{d,2}(t) + v(t)$$

$$f_b(x) = \begin{cases} \begin{bmatrix} 1 & 0 \end{bmatrix}^T \text{, if } \left\| x - \begin{bmatrix} 1 & 0 \end{bmatrix}^T \right\| \le \left\| x - \begin{bmatrix} 0 & 1 \end{bmatrix}^T \right\| \\ \begin{bmatrix} 0 & 1 \end{bmatrix}^T \text{, else} \end{cases}$$

$$\begin{bmatrix} x_{d,1}(0) & x_{d,2}(0) & x_{c1}(0) & x_{c2}(0) \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 2 & 0 \end{bmatrix}^T$$

8.1) Case Study: Battery Diagnostics/Prognostics

SOH Estimation Module (Self-recharge Phenomena)

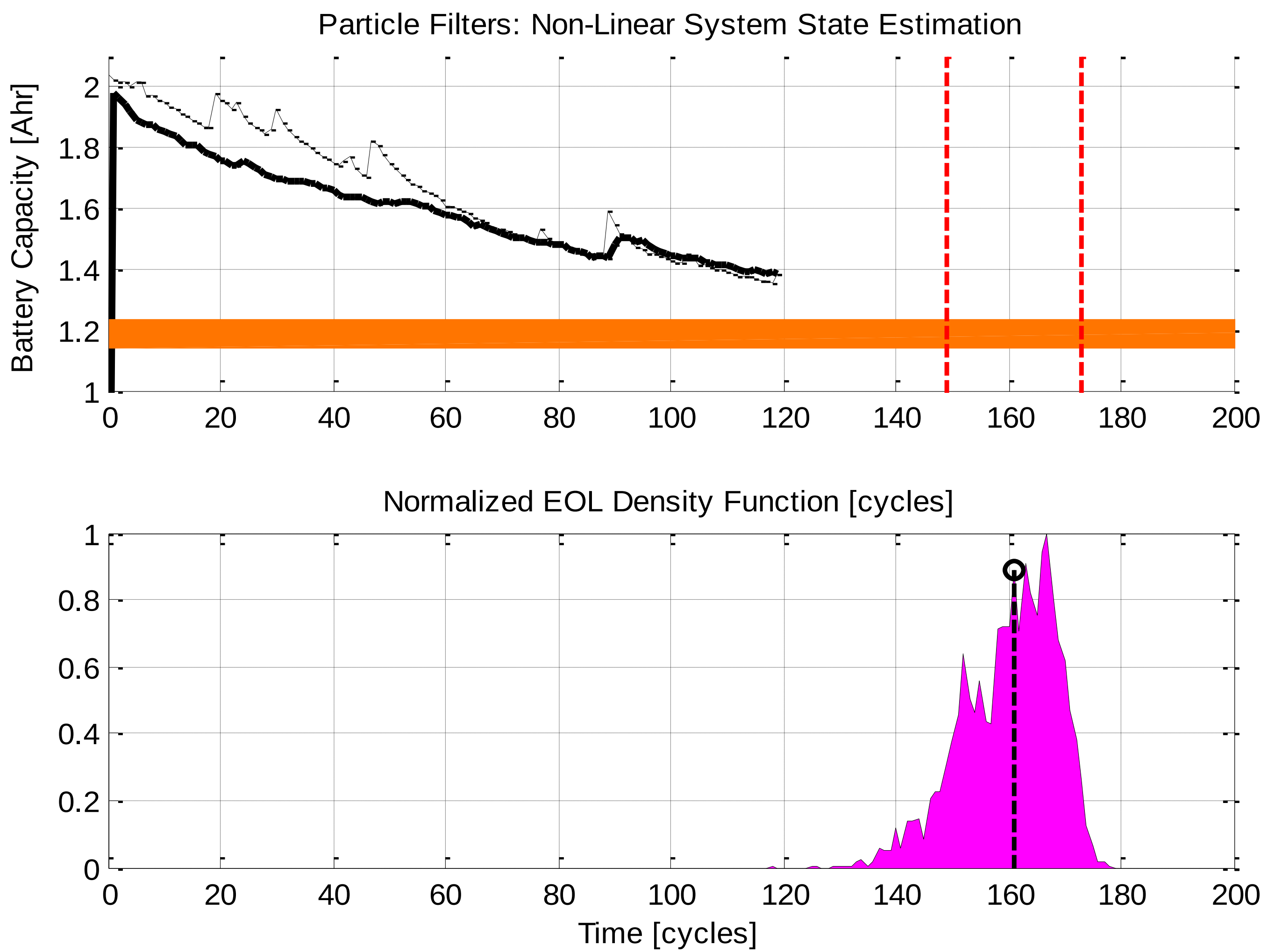
- **State Equation Dynamic Model**

$$\begin{cases} x_1(t+1) = x_1(t) + C \cdot x_2(t) \cdot (a - b \cdot t + t^2)^m + \omega_1(t) \\ x_2(t+1) = x_2(t) + \omega_2(t) \\ x_3(t+1) = \alpha \cdot x_3(t) + \omega_3(t) \end{cases}$$

$$y(t) = x_1(t) + x_3(t) + v(t)$$

- $x_1(t)$ is a state representing the fault dimension
- $x_2(t)$ is a state associated with an unknown model parameter
- $x_3(t)$ is a state associated with the capacity regeneration phenomena
- a, b, C and m are constants associated to the duration and intensity of the battery load cycle (external input U)

8.1) Case Study: Battery Diagnostics/Prognostics



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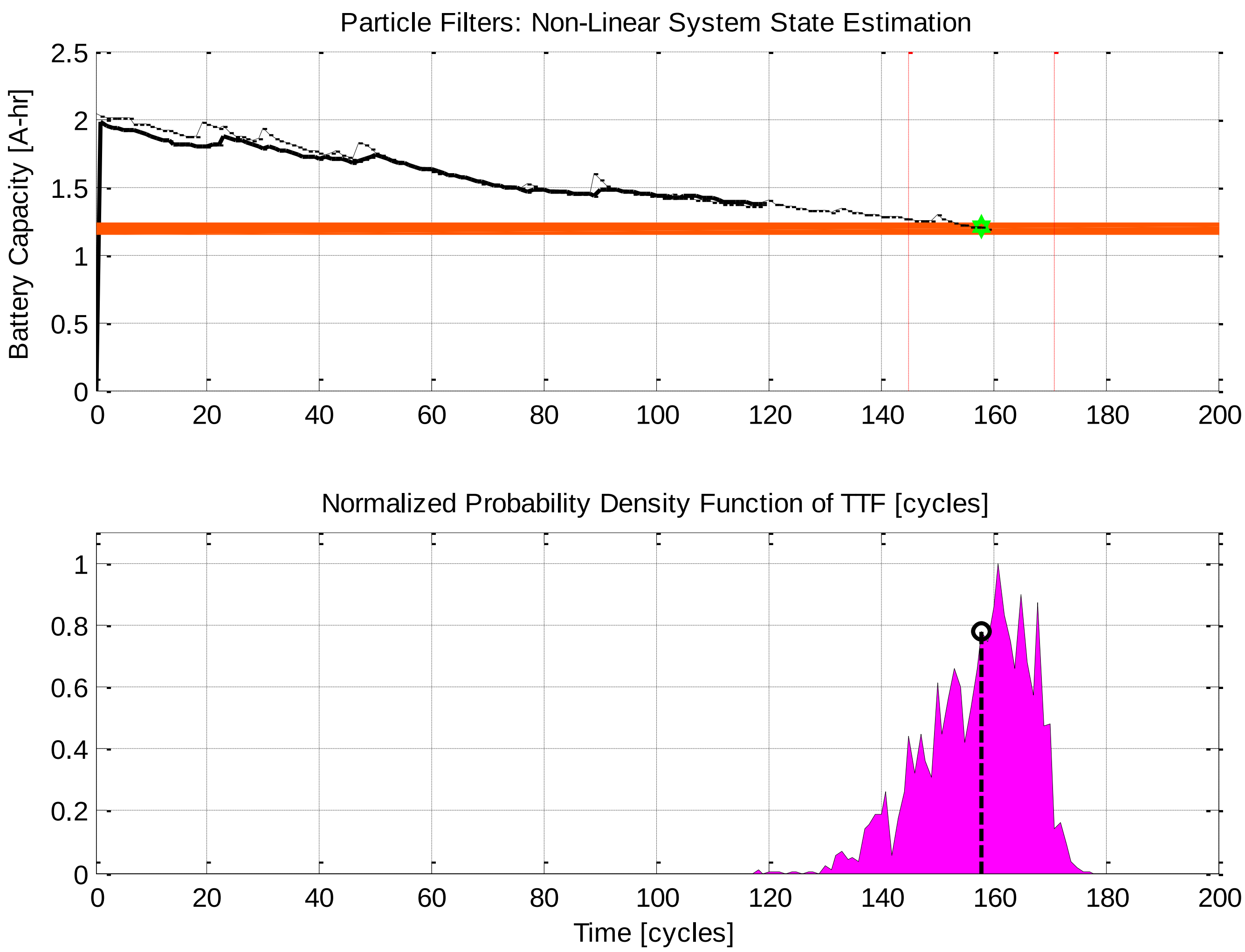
8.1) Case Study: Battery Diagnostics/Prognostics



SOH Estimation Module (Self-recharge Phenomena)

- **State Equation Dynamic Model**

$$\begin{cases} x_1(k+1) = \eta_c x_1(k) + x_2(k) x_1(k) + w_1(k) \\ x_2(k+1) = x_2(k) + w_2(k) \\ x_3(k+1) = \delta(U(k)) \cdot [w_{31}(k)] + \delta(1-U(k)) \cdot [x_3(k) w_{31}(k)] + \delta(2-U(k)) \cdot [x_3(k) + w_{31}(k)] \end{cases}$$

$$y(k) = x_1(k) + [\delta(1-U(k)) + \delta(2-U(k))] x_3(k) + v(k)$$

- η_c is the Coulombic efficiency
- x_1 is a state representing the battery SOH
- x_2 is a state associated with an unknown model parameter
- x_3 is a state associated with the added SOH due to regeneration phenomena
- U is a external input associated with the apparition of regeneration phenomena
- w_1, w_2, w_{31}, w_{32}, and v are iid non-Gaussian noises

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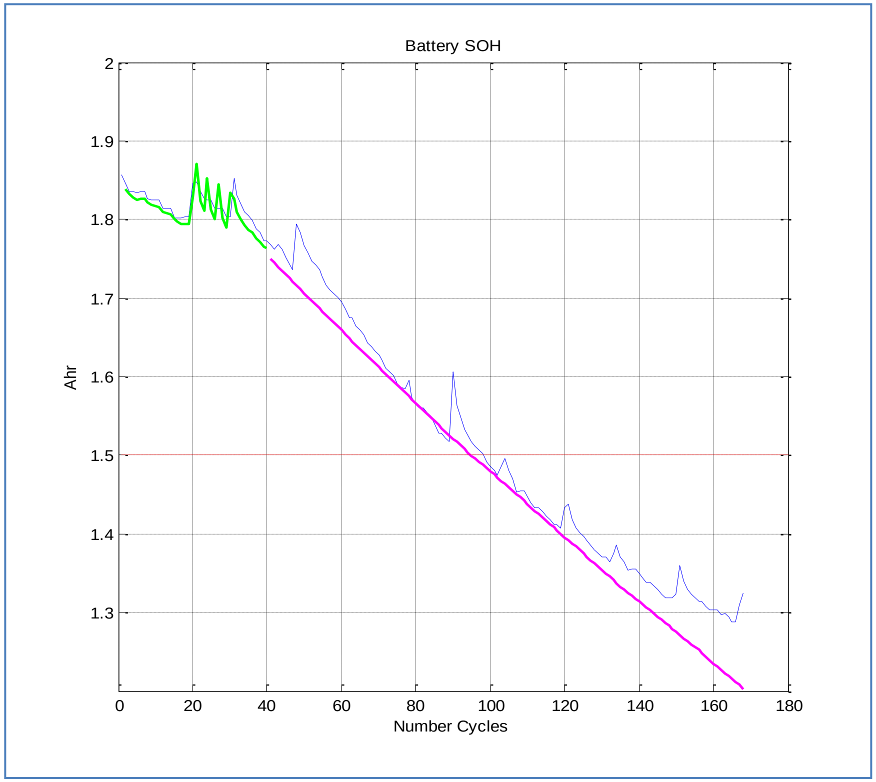

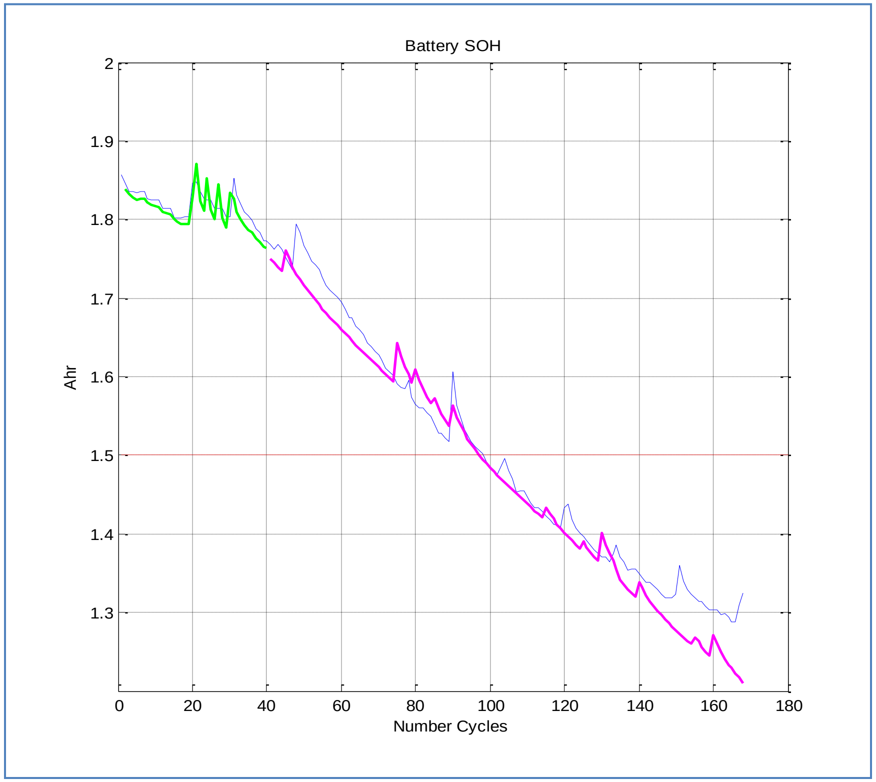

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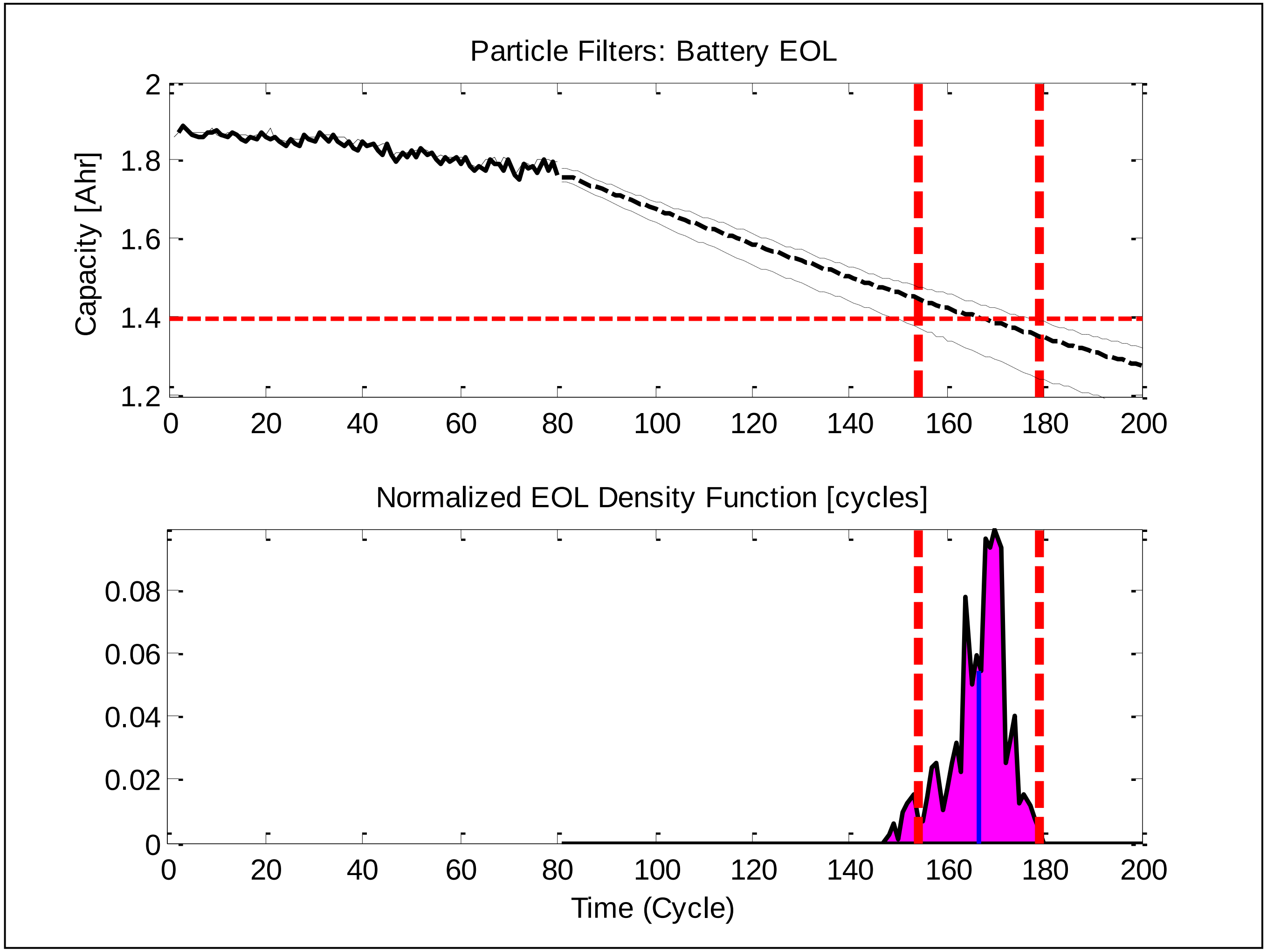

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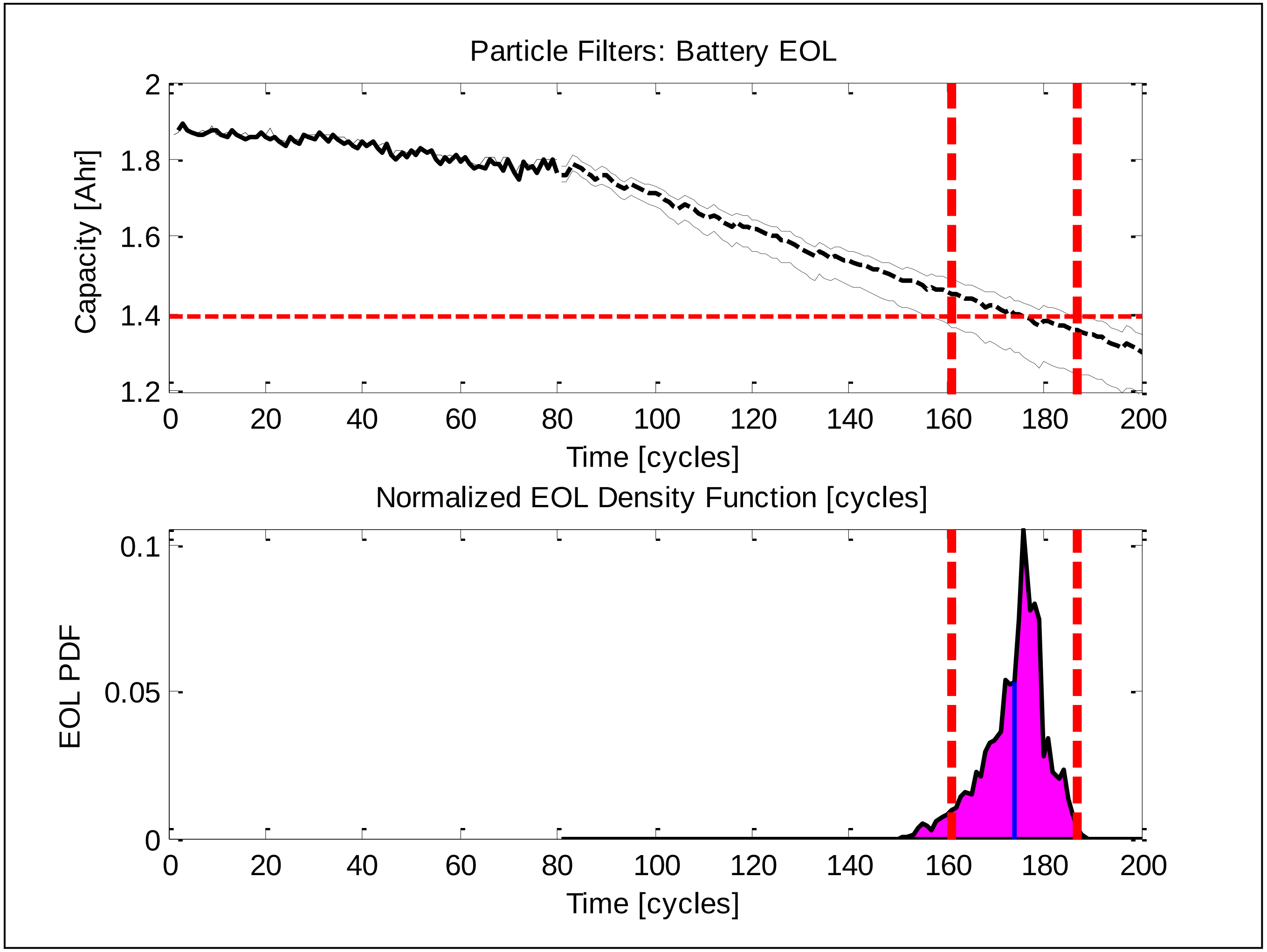

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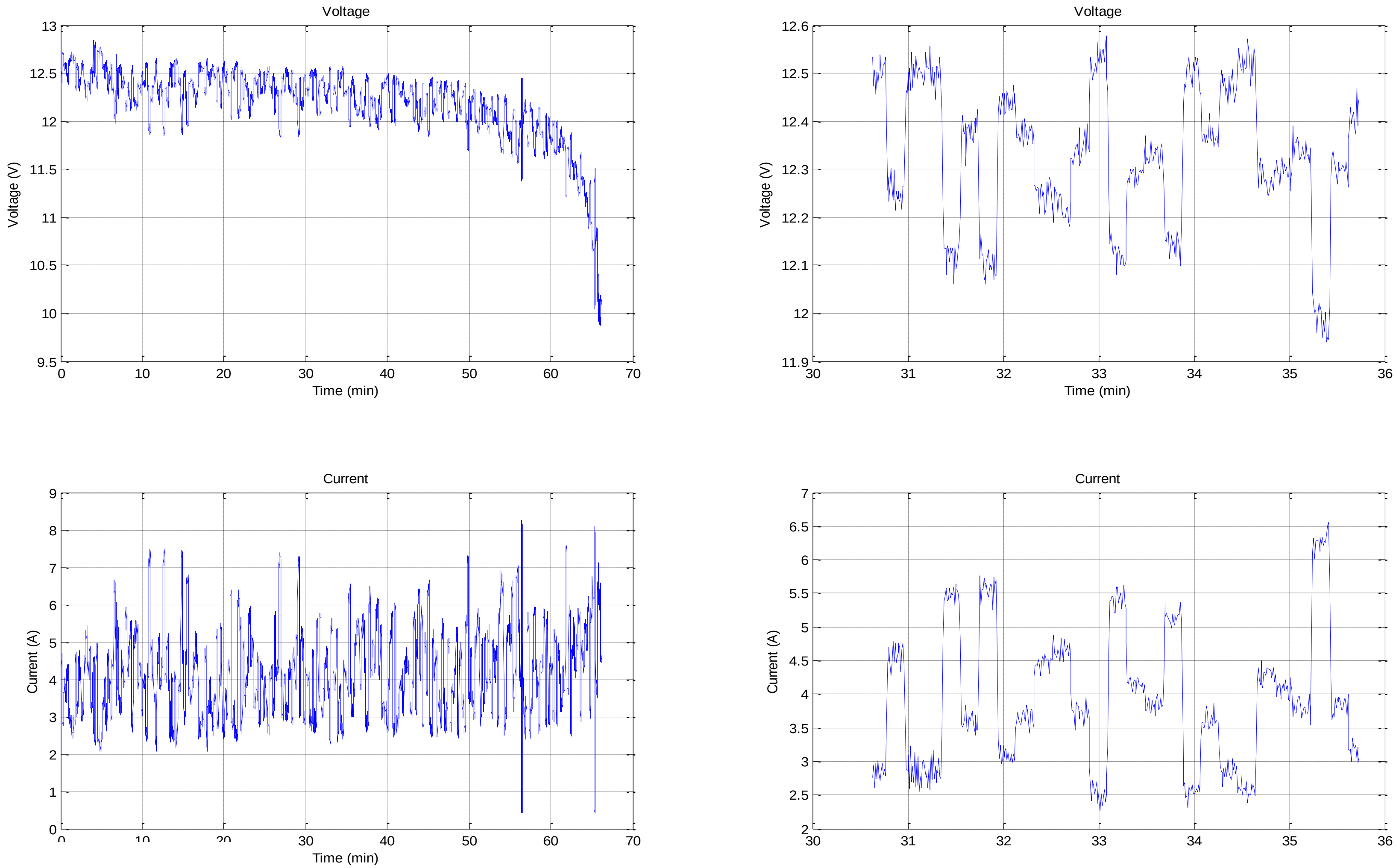
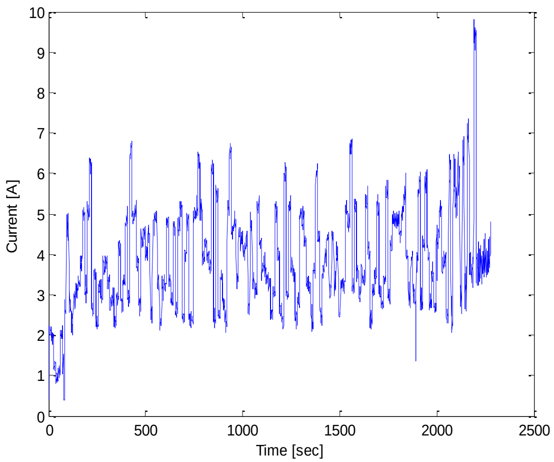
8.2) Case Study: Battery Diagnostics/Prognostics

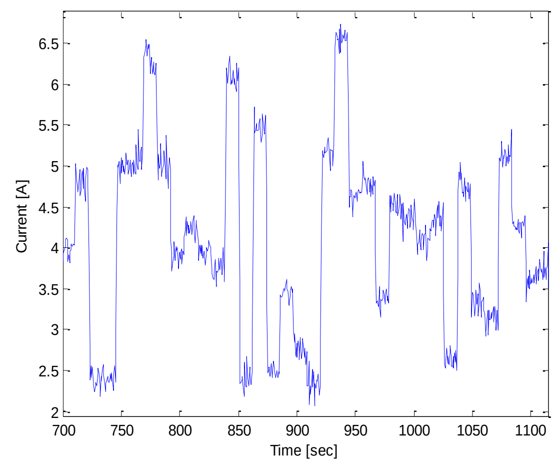
- State-of-Charge Prognosis:



8.2) Case Study: Battery Diagnostics/Prognostics

- State-of-Charge Prognosis:
  - Probabilistic characterization of usage conditions
  - Real-time state estimation/prognosis
  - Self-tuning model (parameter estimation)
  - PF-based framework allows to compute confidence bounds for SOC predictions
  - Modeling the future usage





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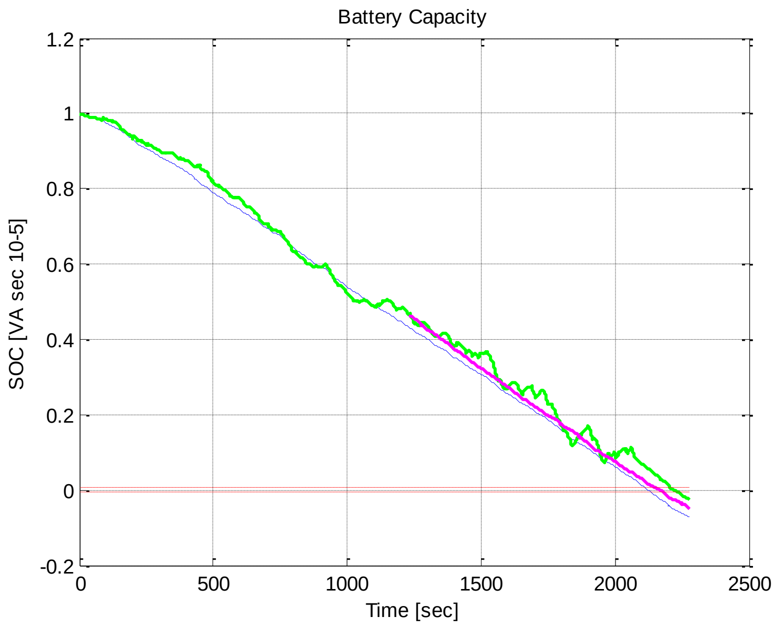
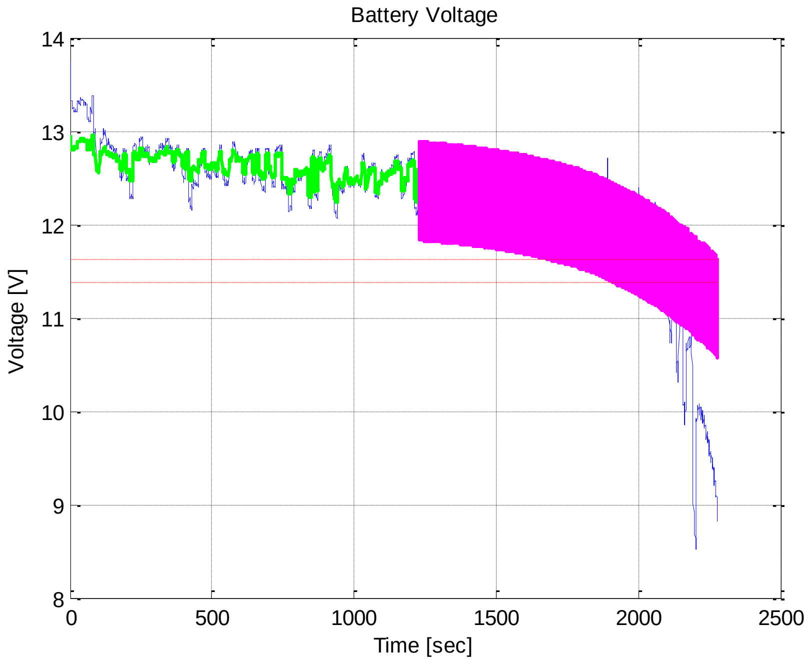
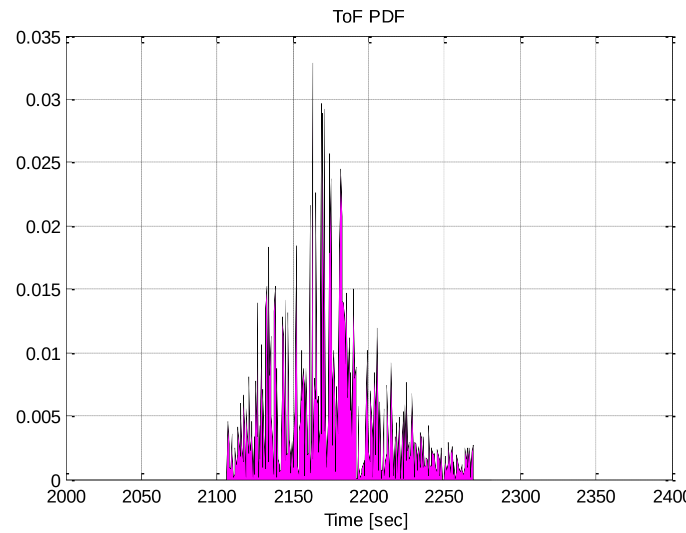
8.2) Case Study: Battery Diagnostics/Prognostics

- State-of-Charge Prognosis:
  (Preliminary Results)







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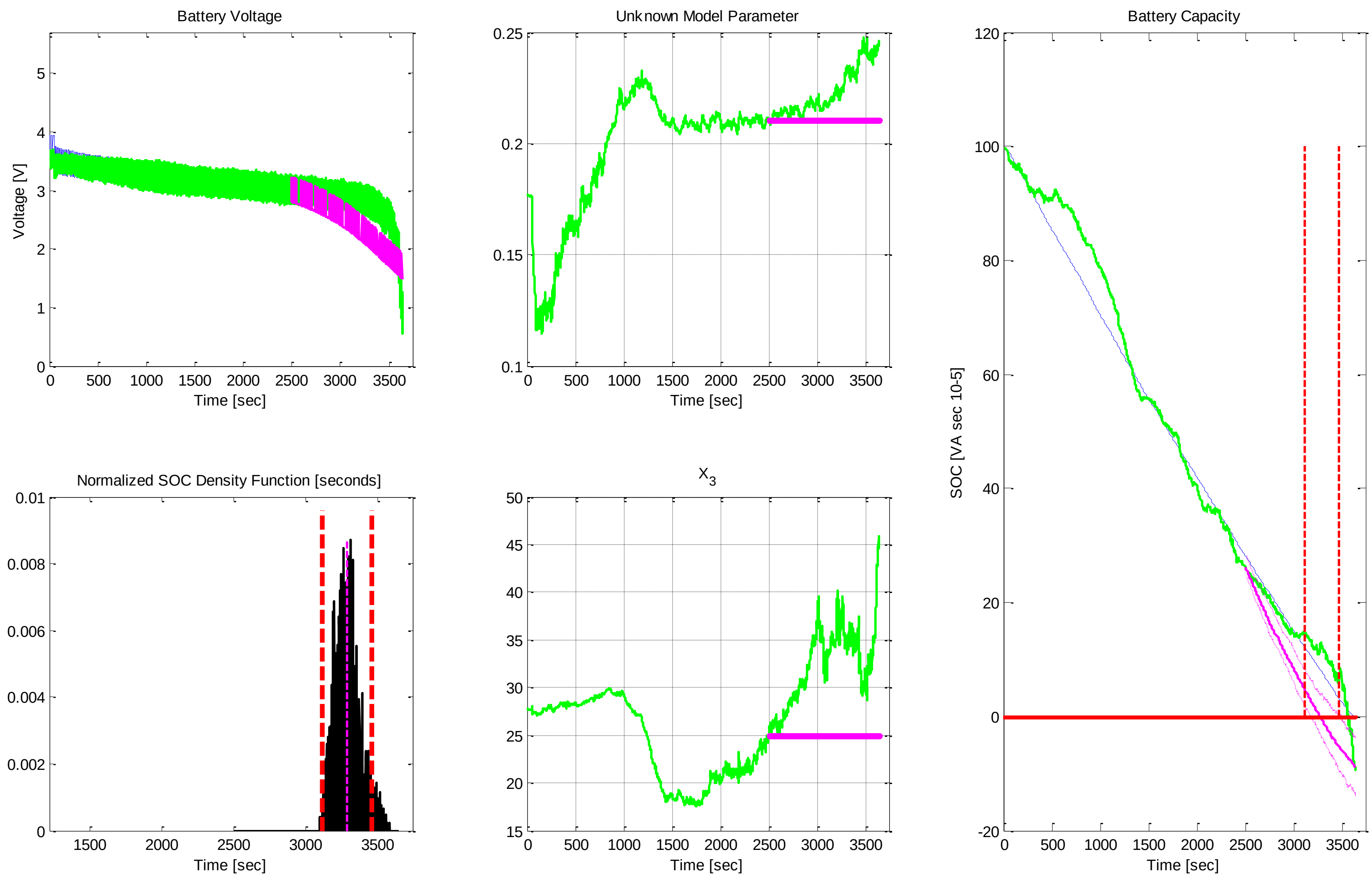
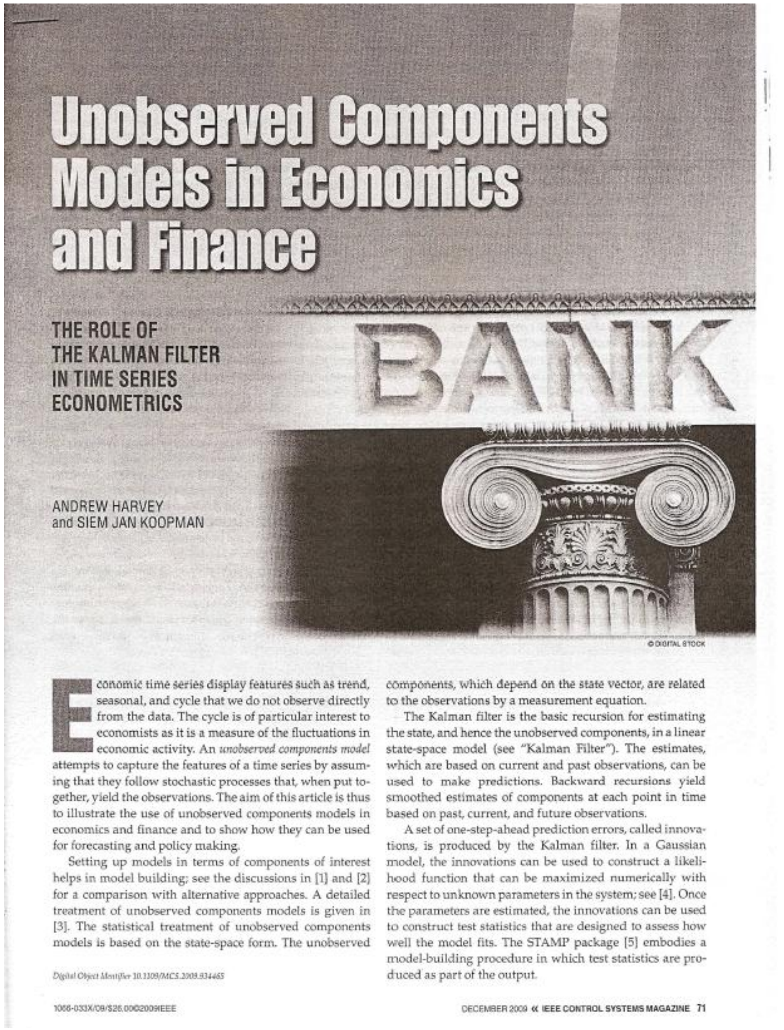
8.2) Case Study: Battery Diagnostics/Prognostics

- State-of-Charge Prognosis: (Preliminary Results)



8.3) Case Study: PF-based Risk Analysis in Finance



Unobserved Components Models in Economics and Finance

THE ROLE OF THE KALMAN FILTER IN TIME SERIES ECONOMETRICS

ANDREW HARVEY and SIEM JAN KOOPMAN

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Economic time series display features such as trend, seasonal, and cycle that we do not observe directly from the data. The cycle is of particular interest to economists as it is a measure of the fluctuations in economic activity. An *unobserved components model* attempts to capture the features of a time series by assuming that they follow stochastic processes that, when put together, yield the observations. The aim of this article is thus to illustrate the use of unobserved components models in economics and finance and to show how they can be used for forecasting and policy making.

Setting up models in terms of components of interest helps in model building; see the discussions in [1] and [2] for a comparison with alternative approaches. A detailed treatment of unobserved components models is given in [3]. The statistical treatment of unobserved components models is based on the state-space form. The unobserved components, which depend on the state vector, are related to the observations by a measurement equation.

The Kalman filter is the basic recursion for estimating the state, and hence the unobserved components, in a linear state-space model (see "Kalman Filter"). The estimates, which are based on current and past observations, can be used to make predictions. Backward recursions yield smoothed estimates of components at each point in time based on past, current, and future observations.

A set of one-step-ahead prediction errors, called innovations, is produced by the Kalman filter. In a Gaussian model, the innovations can be used to construct a likelihood function that can be maximized numerically with respect to unknown parameters in the system; see [4]. Once the parameters are estimated, the innovations can be used to construct test statistics that are designed to assess how well the model fits. The STAMP package [5] embodies a model-building procedure in which test statistics are produced as part of the output.

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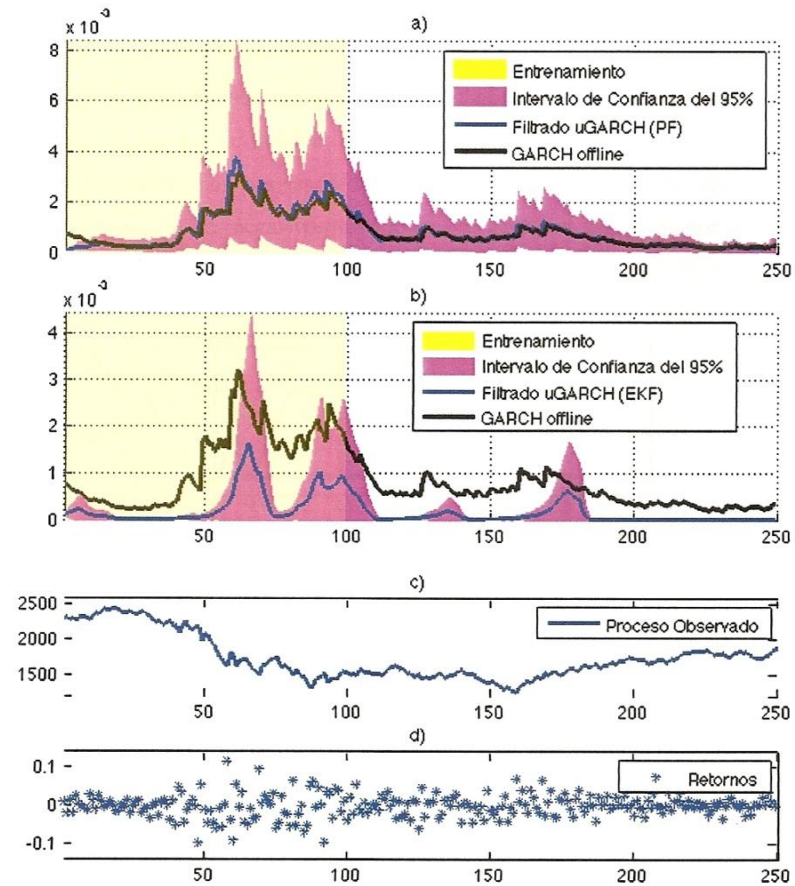
$$u'_t = \sigma_t \eta_t$$
$$\eta_t \sim \mathcal{N}(0,1) \text{ i.i.d } \forall t.$$

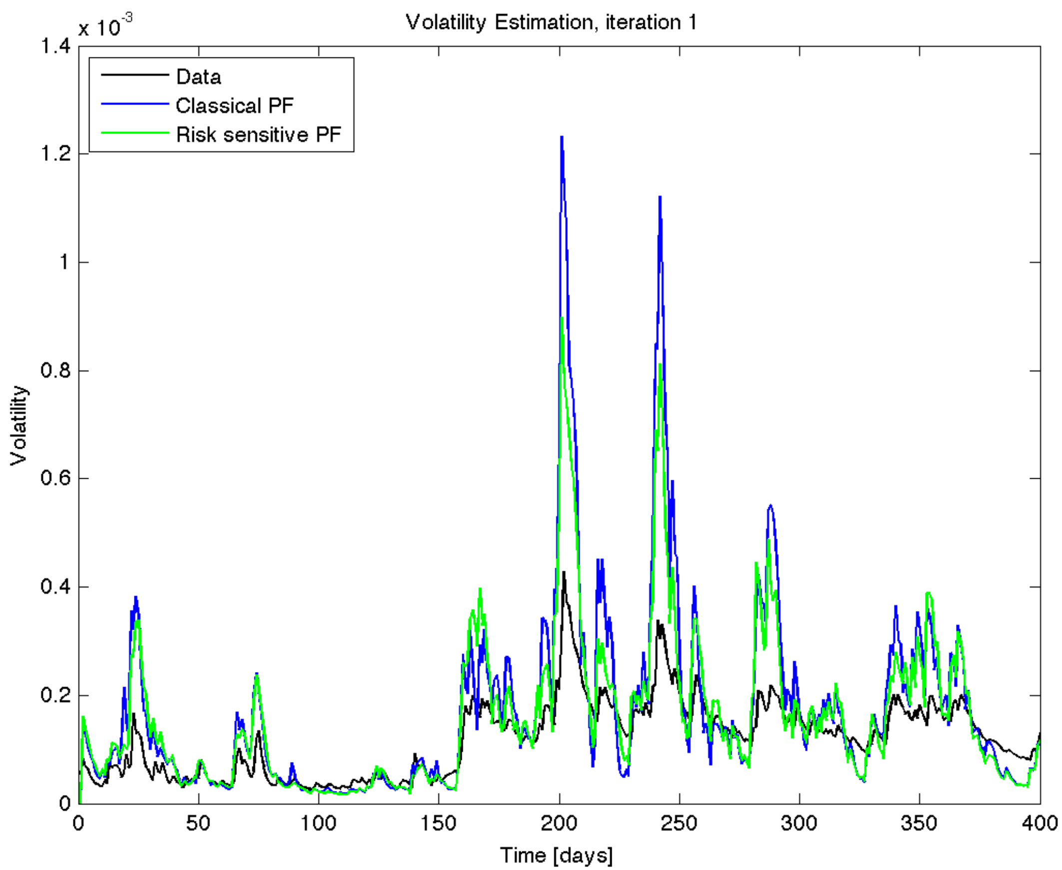
$$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2\eta_{t-1}^2 + \beta\sigma_{t-1}^2$$
$$r_t = \mu + \sigma_t \epsilon_t.$$



8.3) Case Study: PF-based Risk Analysis in Finance

$$\sigma_t^2 = \omega + \alpha\sigma_{t-1}^2\eta_{t-1}^2 + \beta\sigma_{t-1}^2$$
$$r_t = \mu + \sigma_t\epsilon_t$$

- r_t: Return process
- σ_t: Stochastic volatility
- $\mu \in \mathbb{R}$
- $\omega \in \mathbb{R}^+$
- α, β: Parameters in $[0,1]^2$
- $\epsilon_t \sim \mathcal{N}(0,1)$
- $\eta_t \sim \mathcal{N}(0,\sigma)$



Thank You!

Questions?

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